Software development: a personal and historical view

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  - An overview of how the mathematical software available to me has developed over the last 40 years: I will try to show how "you have never had it so good" — the software and related tools which we all have at our fingertips today, for free, enables us to be vastly more productive than I would have dreamed possible 40 years ago.

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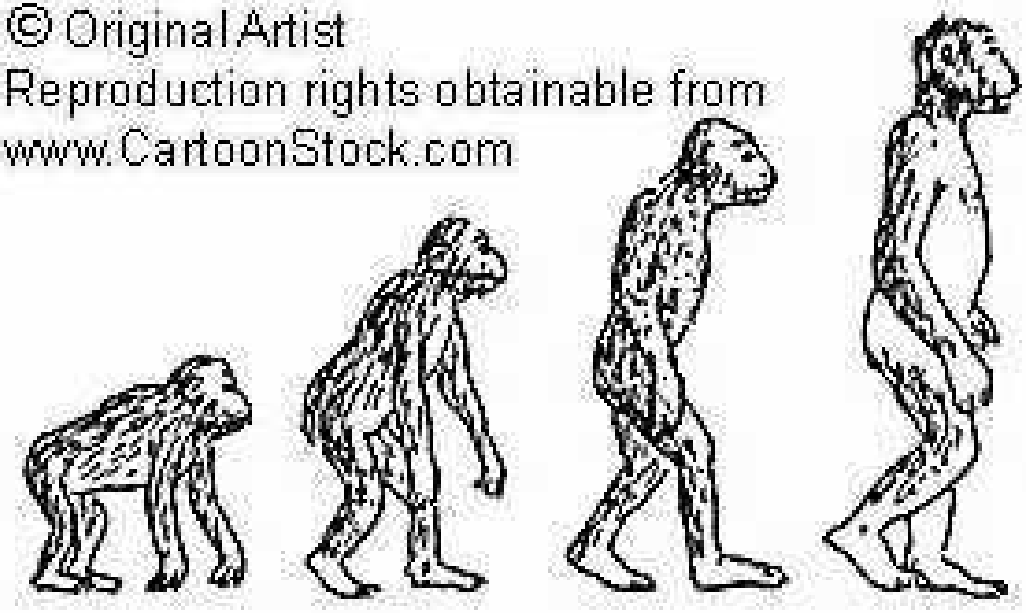
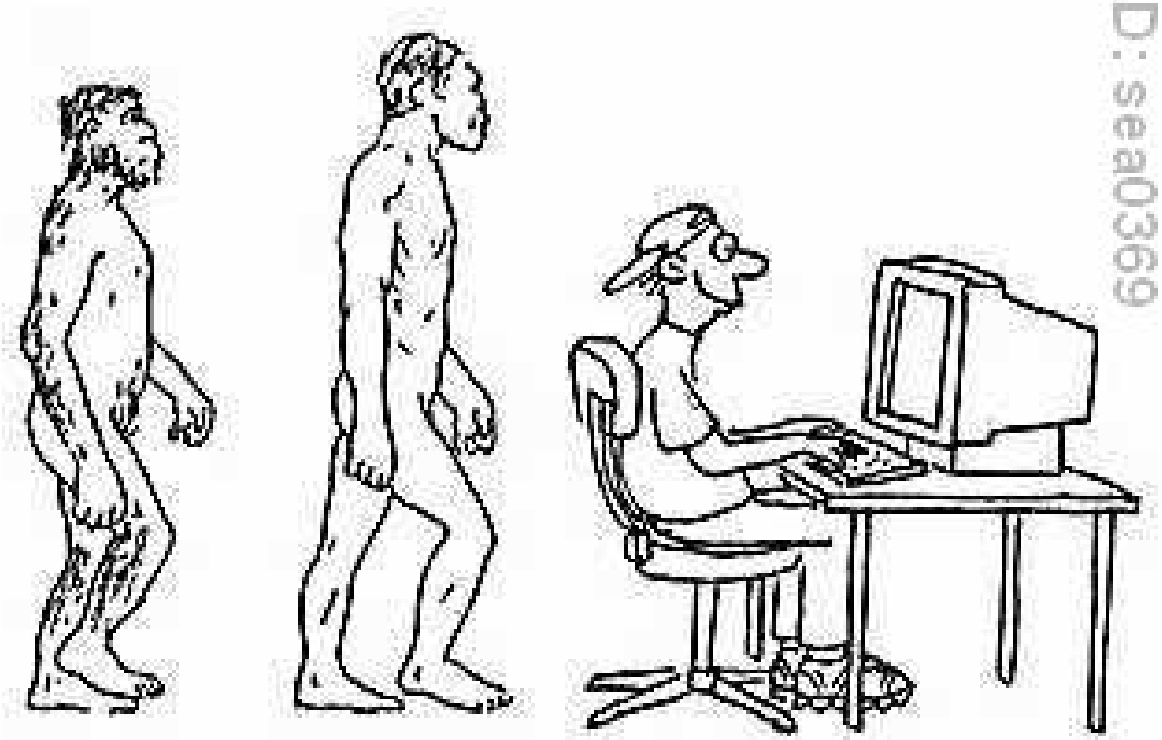
Ask questions!





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- I got quite good at finding primes. Integers were 16-bit (up to 32767) but I got up to about 108,000. I kept the printout (the only output of the only run) on my wall for years.

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- It had a core storage: if you looked into the box you could see the tiny metal rings threaded onto wires, storing one bit each.

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- Exercise: do this in `SageMath` in the next five minutes!

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- ...still in `FORTRAN`.
- I also had to debug programs written by other people who had left. This involved spending hours staring at `FORTRAN` programs full of conditional GOTO statements. Not nice.

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- Since no-one had computed any of these before (except for degree 1 prime levels) no-one actually knew they were wrong! (And the bugs were fixed later.)

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- The mathematics graduate students who computed were split between number theorists (Birch students) and graph theorists (Cameron students), who used `Algol68`, and the rest (mostly applied mathematicians) who used `FORTRAN`.

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- When Richard read a draft of my 1992 book "Algorithms for Modular Elliptic Curves" he noticed that the variable names in my description of Tate's Algorithm we exactly those from his 1980 `FORTRAN` program. And the same variable names are used in both `SageMath`'s and `Magma`'s implementations – which gives you a clue as to who wrote them!

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- It was there that for the first time ever I computed (by hand) modular symbols for $\Gamma_0(11)$ and also (by hand) several Hecke eigenvalues, and hence (using the Apple II) the periods $(1.2692, 0.6346 + 1.4588i)$ and $c_4 = 496$, $c_6 = 20008$ of the "first" elliptic curve `11a1`.

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- Programming the Apple II in `Pascal` was so painful (there were no built-in complex numbers) that I did most of it by hand with a pocket calculator.

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- The machines which ran Pascal and Macsyma were different. I computed the matrices of Hecke operators in Pascal on one machine, and used some kind of ftp to transfer files to the other machine where I used Macsyma to find their eigenvalues.

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  - I discovered C++.

Why C++?

- To me, moving to C looked like a move backwards; I was used to having user-defined classes, and overloaded operators, for example. I want to be able to write

$$a = b + c$$

  and not

$$\texttt{xyz\_add}(a, b, c)$$

  (where a, b, c may be complex numbers or vectors or matrices or points on an elliptic curves, or whatever).
- I wanted to use a mainstream language which others used:
  - –so that others could use the programs I wrote
  - –so that I could make use of libraries written by others

LiDIA, NTL and PARI

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LiDIA RIP

- I contributed elliptic curve code to LiDIA, but found that they did not have a good mechanism for people outside Buchmann's own research group to contribute code. It was common to get no response: the LiDIA group had some very competent people in it, but they were all on short-term contracts (graduate students and postdocs) whose main responsibility was (reasonably!) to their own work. And Buchmann himself was not involved at all in writing code; he was more of a figurehead.

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- And there are now alternatives!

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- There are many useful GP scripts available on many mathematicians' web pages. These have varying quality, and are written in various different dialects of the GP language. Some mathematicians[2] use GP *a lot*, (and no other software), and still use a version now 20 years old!

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  This is very similar to using Cython to speed up Python.

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- Relatively few people work on Pari development: mostly KB, BA and Jeroen Demeyer (from SageMath).
- The heavy use of Pari in SageMath has revealed many bugs in Pari, all of which have been rapidly fixed upstream.

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- SageMath relies on Pari entirely for all its number field functionality (with very few exceptions).
- In turn, SageMath users greatly increase the Pari user base, and so SageMath (and all the relevant doctests) provides a great platform for testing Pari (finding bugs, occasionally fixing them, helping to improve the build system).

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SageMath also has a free online version called CoCalc (used to be SageMathCloud). See http://cocalc.com.

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- The feeling that the code I write will *last*: instead of rotting away, becoming harder and then impossible to use, it will remain available for ever (or until replaced by improved code).

Other systems and packages

Time has not permitted me to discuss all relevant software, even that used in number-theory.I have concentrated on the ones I know best. There are around 75 items listed at `http://www.numbertheory.org/ntw/N1.html`, including all those mentioned here, several which are components of `SageMath`, and more.

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I will finish by mentioning a few of these.

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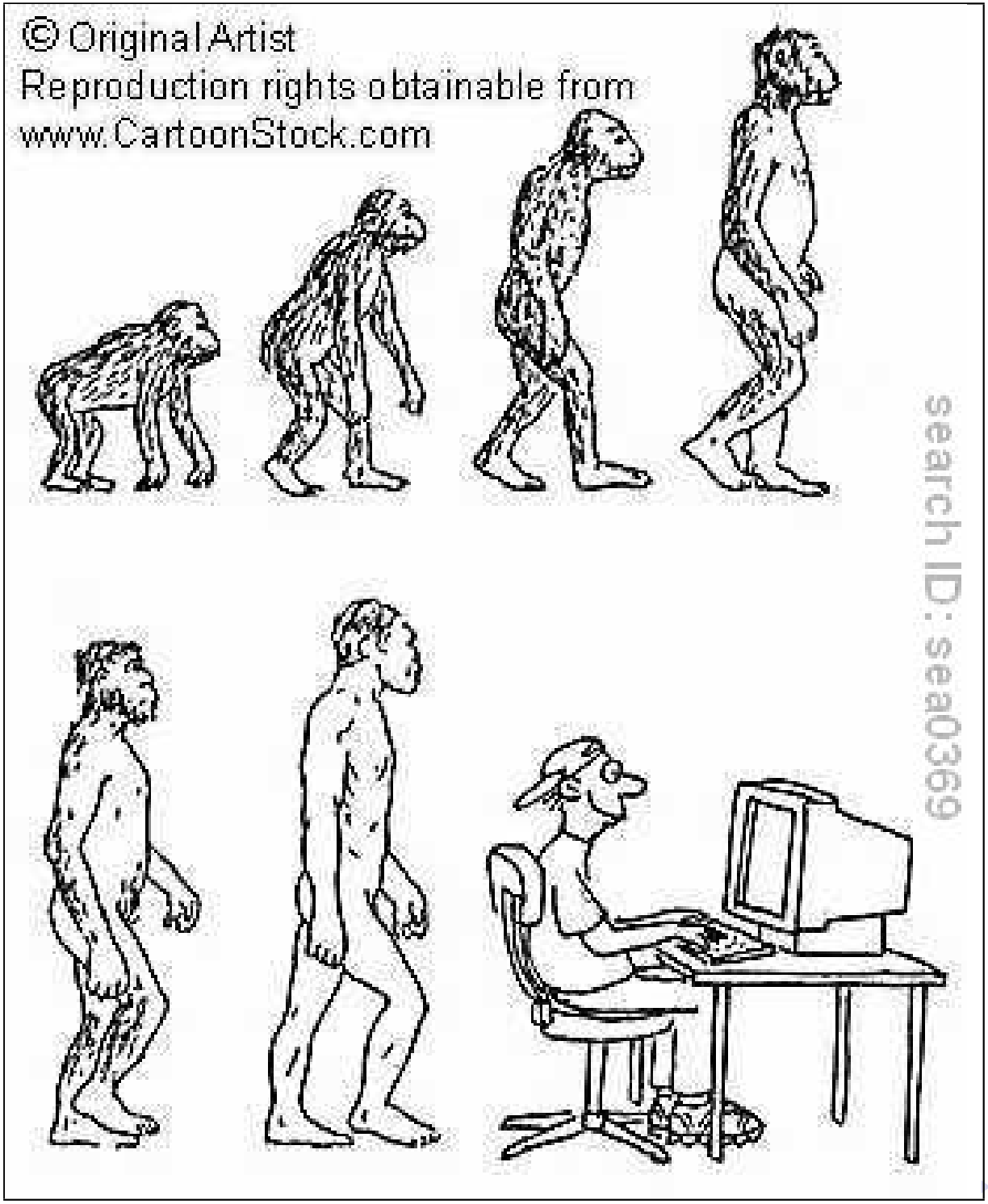
- `SiMATH`, developed by Horst Zimmer in Saarbruecken with the help of many of his students and assistants; funded by Siemens, its code was never open and is now lost, along with a vast amount of human effort.
- `Macaulay2` for algebraic geometry
- ...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