HOMEWORK 2, MATH 171, SPRING 2003

DUE THURSDAY APRIL 17

(1) **Decimal Expansion** Let r be a real number. We define the decimal expansion recursively as follows: Let $d_0 \in \mathbb{Z}$ be the largest integer less than r.

If $d_0, \ldots, d_k$ have been defined, let $r_k = \sum_{l=0}^{k} d_l/10^l$, and let $r' = r - r_k$. Let d_{k+1} be the largest integer less than $10^{k+1}r'$.

  (a) Determine $d_0, d_1, \ldots$ for $r = 1/2$ and $r = -1/3$.
  (b) Show that d_k exists for all $k > 0$. (ie, show that there is a largest integer less than r or $10^{k+1}r'$.)
  (c) Show that $0 \leq d_k \leq 9$ for $k > 0$.
  (d) Show that the sequence $r_0, r_1, r_2, \ldots$ converges to r.
  (e) Show that if $c_0, c_1, c_2, \ldots$ is a sequence of integers with $0 \leq c_k \leq 9$ for $k > 0$ then the sequence $r_k = \sum_{l=0}^{k} c_l/10^l$ converges. Let r be its limit. Show that either there is some $N > 0$ for which $c_l = 0$ for all $l > N$ or $c_l = d_l$ for all $l \geq 0$, where $d_0, d_1, d_2, \ldots$ is the sequence obtained as above from the real number r.

(2) From the textbook: Section 1.2, #3, page 45

(3) Section 1.7, #1, page 70

(4) Exercises at the end of Chapter 1 (pages 97-102), #10