MATH 111 HOMEWORK 7, WINTER 2004

DUE FRIDAY, MARCH 12

(1) Write an algorithm to intersect two monomial ideals I and J. You should prove that your algorithm works, and give an example.
(2) State and prove criteria for when $IJ = I + J$, where I and J are monomial ideals.
(3) Show that a monomial ideal is prime if and only if it is generated by a subset of the variables.
(4) Recall that a monomial ideal I is radical if and only if it has a squarefree generating set. Give an algorithm to compute monomial prime ideals $P_1, \ldots, P_s$ such that $I = \cap_{i=1}^{s} P_i$. You should prove that your algorithm works, and illustrate it on the ideal $I = \langle x_1x_5, x_1x_3, x_3x_4 \rangle$. Does your algorithm require k to be algebraically closed?