MATH 111 HOMEWORK 1

DUE 1/16/04

You are encouraged to work together on the exercises. Any graded assignment, though, should represent your own work. References to CLO refer to the textbook, which is on reserve at the Mathematics library (on the fourth floor of Building 380).

(1) (a) Reread the definition of a field (for example, in Appendix A from the text, or from the 120 text, or from a book in the library).
   (b) For a prime p, let $\mathbb{F}_p$ be the field whose elements are $\{0, 1, \ldots, p-1\}$, with addition and multiplication modulo p. Show that $\mathbb{F}_p$ is a field.

(2) (CLO Chapter 1, Section 4, #1) Consider the equations:

$$\begin{aligned} x^2 + y^2 &= 1 \\ xy - 1 &= 0 \end{aligned}$$

which describe the intersection of a circle and a hyperbola.
   (a) Use algebra to eliminate y from the above equation.
   (b) Show that the polynomial you found in the first part lies in the ideal $\langle x^2 + y^2 - 1, xy - 1\rangle$.

(3) (CLO, 1.4 #2) Let $I \subseteq S = k[x_1, \ldots, x_n]$ be an ideal, and let $f_1, \ldots, f_s \in S$. Prove that the following statements are equivalent:
   (a) $f_1, \ldots, f_s \in I$.
   (b) $\langle f_1, \ldots, f_s\rangle \subseteq I$.

(4) (CLO, 1.4 #3) Use the previous exercise to prove the following equalities of ideals in $k[x, y]$.
   (a) $\langle x + y, x - y\rangle = \langle x, y\rangle$.
   (b) $\langle x + xy, y + xy, x^2, y^2\rangle = \langle x, y\rangle$.
   (c) $\langle 2x^2 + 3y^2 - 11, x^2 - y^2 - 3\rangle = \langle x^2 - 4, y^2 - 1\rangle$.

(5) MathSciNet (`www.ams.org/Mathscinet`) indexes the vast majority of mathematics papers and books published each year. Use it to find out:
   (a) How many other books have the authors of our text-book written? Does our library have any of them?

(b) For how many papers published between 1990 and 1995 does the word "Gröbner" appear in the review?

You will need to be logged on to a computer on the Stanford network to use MathSciNet.

(6) In this class we will use the software package `Macaulay 2` extensively. This is a free package, and runs on Windows, Macs, and many flavors of Unix. Go to: `http://www.math.uiuc.edu/Macaulay2/` and download a copy - either to your own computer, or a Stanford account (such as on the Stanford system `tree.stanford.edu`). Before doing this, you should check that Macaulay 2 is not already installed on any system you have access to.

When you have installed the package, test it by typing the following at the command line:

```
R=QQ[x,y,z]
```

followed by

```
I=ideal(x^2-y,x^3-z,x*y-z)
```

and finally

```
(x*z-y^2)%I
```

Hand in your output as an answer to this question.