MATH 108 HOMEWORK 7

DUE THURSDAY, DECEMBER 5

(1) Let n be an even positive integer. Write the numbers $1, 2, \dots, n^2$ in the squares of an $n \times n$ grid so that th kth row, from left to right, is $(k-1)n + 1, (k-1)n + 2, \dots, (k-1) + n$. Color the squares of the grid so that half of the squares in each row and in each column are red and the other half are black (a checkerboard coloring is one possibility). Prove that for each such coloring, the sum of the numbers on the red squares is equal to the sum of the numbers on the black squares. (Hint: Think back several weeks.)

(2) Let P be the poset whose elements are subspaces of the vector space $\mathbb{F}_p^d$, where $x \leq y$ if x is a subspace of y. Show that P is lattice. Show that every maximal chain in P has the same length. What is that length? How many subspaces are there of dimension k?

(3) 25A(i) (you can assume Theorem 25.1).