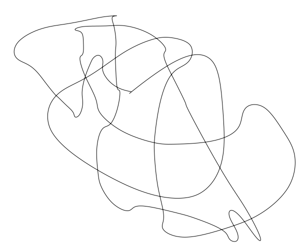
MATH 108, FALL 2002

HOMEWORK 1

This homework is due on Tuesday, October 8, in class. No late homework will be accepted. You are encouraged to work together, but must write up the answers individually. Be sure to provide proofs of all your answers.

(1) 1C
(2) 1D
(3) 1G
(4) 1H
(5) 2A
(6) **Doodles:** Draw a closed squiggle on the plane. It can cross itself many times (including at the same point) but cannot run tangent to itself. The squiggle divides the plane into several regions. Show that you can colour these regions with two colours so that no adjacent regions have the same colour.



(7) **Ramsey Theory:** Show that if there are six people at a party, we can either find three all of whom know each other, or three all of whom do not know each other. What if six is replaced by five? What bounds can you give on what six needs to be replaced by if three is replaced by four? (You do not need to give the lowest possible number, but must justify your answer).
(8) **Parking Functions:** Imagine n cars want to park on a one-way street with n parking spaces. They each have a favourite parking place, and will drive along without looking until they get there. If that space is full, they will park in the next available parking space.
   (a) Let $\phi : \{1, \dots, n\} \to \{1, \dots, n\}$ be the function which records the space choice of each car, so $\phi(i)$ is the number of the space where the ith car would like to park. Show that all the cars can park if and only if for each i we have $|\{j : \phi(j) \leq i\}| \geq i$. Such functions are called *parking functions*.
   (b) Show that there are $(n+1)^{n-1}$ parking functions for n cars.