MACAULAY2 EXERCISES

DIANE MACLAGAN

Remember that `viewHelp` (or `help` if you do not have your browser integrated properly) is a very useful command!

If you don't have Macaulay2 installed, you can use the web interface. You will also want to (eventually) read the tutorials on the M2 webpage, starting with the first one (take a look if you finish the exercises fast!)

(1) Write a function isEven that takes as input an integer and returns 1 if it is even, and 0 if it is odd. A useful command is `%` :

```
i1 : 5 % 3

o2 = 2
```

Use `==` to test equality.

(2) The ideal I showed during the presentation is the ideal of the image of the 3-uple Veronese embedding of $\mathbb{P}^1$ into $\mathbb{P}^3$. We will now compute this in another way.
   (a) Create a polynomial ring R in two variables.
   (b) Create a list of the degree 3 monomials in your ring. The command `basis(3,R)` will help, as will the commands `entries` and `flatten`.
   (c) Create a polynomial ring in four variables S.
   (d) Create a homomorphism from S to R that sends the ith generator of S to the ith element of your list. The syntax for this is `map(R,S,yourList)`.
   (e) Compute the kernel of this map. The relevant command is `kernel`. Compare this with the ideal in presentation.

(3) Compute a Gröbner basis for the ideal $\langle x^3y^2 - 4x^2y^3 + 5y^5, x^6 - 7xy^5\rangle \subseteq \mathbb{Q}[x, y, z]$. Is $xy^9 \in I$? (Use `%` again).

(4) In this question you will check the equations for the Grassmannian. The Grassmannian $\mathrm{Gr}(d, n)$ is a variety that parameterises all d-dimensional subspaces of an n-dimensional vector space.
   (a) Create a 2×4 matrix with generic entries (e.g., x_{ij}).

(b) Compute the six 2×2 minors of your matrix. The command `gens minors(2,A)` will produce a matrix with these entries.

(c) Compute a homomorphism from a polynomial ring in six variables to your ring that takes the ith generator to the ith minor on your list.

(d) Take the kernel of your homomorphism. This is the ideal of the Grassmannian $\mathrm{Gr}(2,4)$. If you already knew what this variety was, compute the dimension to check that this is correct.

(e) Now write a function that takes as input your choice of $d < n$ to replace 2 and 4.

(f) This command actually already exists in Macaulay2! Look at the help for `Grassmannian`. (This command uses the projective convention for the Grassmannian, so to see our example you should type `Grassmannian(1,3)`). How can you use this to test that your function is correct?

(5) Write a method `isSingular` that decides if a matrix or the affine variety defined by an ideal is singular. For the variety, you may consider the variety of an ideal in

```
QQ[x_1..x_n]
```

as living in $\mathbb{A}^n_{\mathbb{C}}$.