INTRODUCTION TO SCHEMES - HOMEWORK 7

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As before, you do no need to do all these exercises for credit, but are expected to choose at least three that are challenging to you at your level.

If you think you have discovered a mistake in one of these exercises, please email me as soon as possible.

(1) (Question changed) Recall that the support of sheaf $\mathcal{F}$ is the set $\{p \in X : \mathcal{F}_p \neq 0\}$. Let I be an ideal in a ring R, and let $M = R/I$. What is the support of the sheaf $\tilde{M}$ on $\text{Spec}(R)$?
(2) Let M be an R module. Show that $\tilde{M}$ is a sheaf on the base $\{D(f)\}$ of $\text{Spec}(R)$.
(3) Let $\phi\colon \mathcal{F} \to \mathcal{G}$ be a morphism of sheaves with both $\mathcal{F}$ and $\mathcal{G}$ quasicoherent. Show that $\ker(\phi)$ is a quasicoherent sheaf.
(4) Let $\phi\colon X \to Y$, and let $\mathcal{F}$ be a quasicoherent sheaf on a Noetherian scheme X. Show $\phi_*\mathcal{F}$ is a quasicoherent sheaf on Y.
(5) Hartshorne II 5.2. (sheaves of modules over DVRs)
(6) Hartshorne II 5.7 (locally free sheaves)
(7) Hartshorne II 5.8 (upper semicontinuity of fibre dimension)
(8) Hartshorne II 5.10 (subschemes of projective space)