INTRODUCTION TO SCHEMES - HOMEWORK 6

DIANE MACLAGAN

As before, you do no need to do all these exercises for credit, but are expected to choose at least three that are challenging to you at your level.

If you think you have discovered a mistake in one of these exercises, please email me as soon as possible.

(1) Show that if $\phi\colon R \to S$ is a surjective homomorphism of graded rings (so homogeneous elements are taken to homogeneous elements), then there is a corresponding closed embedding $\mathrm{Proj}(S) \to \mathrm{Proj}(R)$.
(2) Vakil 6.4E (taking Veronese subrings does not change Proj).
(3) Let $R = \mathbb{C}[x^2, xy, y^2, xz, yz, z^2]$, $S = \mathbb{C}[x^2, xy, y^2, xz, yz]$. Show that $\mathrm{Proj}(R) \cong \mathbb{P}^2$, and $\mathrm{Proj}(S)$ is the blow-up of $\mathbb{P}^2$ at a point (see Ch 1 Hartshorne, or many other references if you do not know what the blow-up is). This means that we have a natural map $\mathrm{Proj}(S) \to \mathrm{Proj}(R)$, even thought we have an inclusion of rings $S \to R$.
(4) Recall that the support of sheaf $\mathcal{F}$ is the set $\{p \in X : \mathcal{F}_p \neq 0\}$. Let I be an ideal in a ring R, and let $M = R/I$. What is the support of the sheaf $\tilde{M}$ on $\mathrm{Spec}(R)$?
(5) Let M be an R module. Show that $\tilde{M}$ is a sheaf on the base $\{D(f)\}$ of $\mathrm{Spec}(R)$.
(6) Let $\phi\colon \mathcal{F} \to \mathcal{G}$ be a morphism of sheaves with both $\mathcal{F}$ and $\mathcal{G}$ quasicoherent. Show that $\ker(\phi)$ is a quasicoherent sheaf.
(7) Let $\phi\colon X \to Y$, and let $\mathcal{F}$ be a quasicoherent sheaf on a Noetherian scheme X. Show $\phi_*\mathcal{F}$ is a quasicoherent sheaf on Y.
(8) Hartshorne II 5.2. (sheaves of modules over DVRs)
(9) Hartshorne II 5.7 (locally free sheaves)
(10) Hartshorne II 5.8 (upper semicontinuity of fibre dimension)
(11) Hartshorne II 5.10 (subschemes of projective space)