TCC - HILBERT SCHEMES AND MODULI SPACES - EXERCISE 8

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If you are taking this module for credit write up your solution to one of these questions and email them to me. Aim to do this within two weeks of the lecture. Choose an exercise at the appropriate level for your background; let me know if you are having trouble finding such an exercise.

(1) The *cross ratio* $(p_1, p_2; p_3, p_4)$ of four distinct points in $\mathbb{P}^1$ is the value of $\phi(p_3)$ under the unique automorphism of $\mathbb{P}^1$ that takes p_1, p_2, p_4 in order to $0, \infty, 1$.
   (a) Show that if $p_i = [1 : z_i]$ for $1 \leq i \leq 4$ then the cross ratio is
$$\frac{(z_3 - z_1)(z_4 - z_2)}{(z_3 - z_2)(z_4 - z_1)}.$$
   (b) Show that there are six possible values for the cross ratio if the points are permuted.

(2) Draw the possibilities for stable curves of genus zero with seven marked points, similarly to Figures 1 and 2.

(3) The dual graph of a stable curve has a vertex, labelled by the genus, for each irreducible component, and an edge for each intersection (including self-intersections), and a labelled leaf (sometimes called a half-edge) for each labelled point.
   (a) Draw the dual graphs for $M_{0,6}$ and M_2.
   (b) We can stratify $M_{g,n}$ by the dual graphs. Show that the dimension of a stratum with dual graph Γ has codimension the number of non-leaf edges of Γ. Thus the closure of the set of stable curves whose dual graph has only only one non-leaf edge is one. These loci are the *boundary divisors* on $M_{g,n}$.

(4) We have only scratched the surface of the topic of $M_{g,n}$. Another option for this week is to write a (equivalent of one to two typed pages) summary of another subtopic.

(5) As before, you may also return to another exercise from earlier weeks.