TCC - HILBERT SCHEMES AND MODULI SPACES - EXERCISE 7

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If you are taking this module for credit write up your solution to one of these questions and email them to me. Aim to do this within two weeks of the lecture. Choose an exercise at the appropriate level for your background; let me know if you are having trouble finding such an exercise.

(1) Give an example of a grading $\deg\colon \mathbb{N}^n \to A$ for some abelian group A for which $S_a \cdot S_b \neq S_{a+b}$ for $a, b \in A$.
(2) Show that an ideal $I \subseteq S$ is homogeneous if and only if for all $f \in I$ we have $f_a \in I$, where f_a is a homogeneous component of f: $f = \sum_{a \in A} f_a$ with $f_a \in S_a$ for all $a \in A$.
(3) Fill in the details of the proof of Lemma 5.
(4) Write down more details of the proof that the multigraded Hilbert scheme given in Example 10 equals $\mathrm{Hilb}_P(\mathbb{P}^{n-1})$.
(5) Show that a coarse moduli space is unique, if it exists.
(6) (For those who know elliptic curves well) Show that the j-line is a coarse moduli space for curves of genus one. You may find the hints at the start of Harris-Morrison useful.
(7) As before, it is also acceptable to substitute a previous exercises that you didn't do for this week.