TCC - HILBERT SCHEMES AND MODULI SPACES - EXERCISE 6

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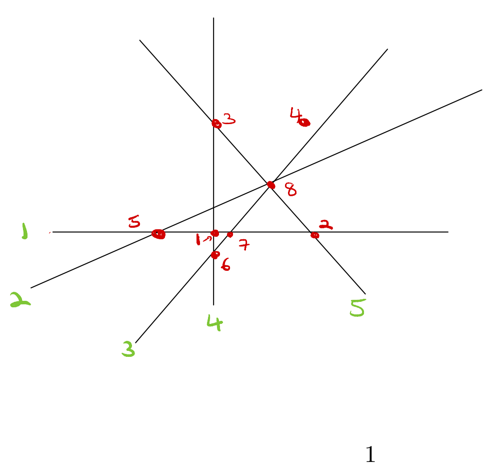
If you are taking this module for credit write up your solution to one of these questions and email them to me. Aim to do this within two weeks of the lecture. Choose an exercise at the appropriate level for your background; let me know if you are having trouble finding such an exercise.

(1) Find a singular point on $\mathrm{Hilb}^5(\mathbb{A}^3)$.

(2) The *radius* of the Hilbert scheme is the radius (length of longest path) of the graph that has one vertex for each irreducible component, and an edge if the components intersect. Reeves (1995) showed that the radius of $\mathrm{Hilb}_P(\mathbb{P}^n)$ is at most $\deg(P)+1$. For Hilbert schemes of points this means that the radius is at most one. Show this directly for $\mathrm{Hilb}^N(\mathbb{A}^d)$. Hints: Gröbner degenerations show that it suffices that show that all monomial ideals live on smoothable component (expand that sentence!) and you can generalise the argument in §2.2 of Lecture 5 for this.

(3) Work through Exercises VI-35, VI-36, and VI-37 of Eisenbud-Harris to understand Mumford's example. (Here I leave the definition of "work through" to your discretion - an expanded sketch of the plan with some details omitted is acceptable, as is consulting other sources).

(4) Compute the point-line incidence scheme for the incidences given in the diagram below.



(5) Follow up on one of the references from this week's lecture notes, and write a one to two page (typed equivalent) summary of the paper. A longer exposition can count as two weeks' problems.
(6) Another option for this week is to return to one of the previous weeks' exercise sheets, and complete a different exercise.