TCC: TROPICAL GEOMETRY

HW 3

(1) Let I be an ideal in $K[x_1^{\pm 1}, \ldots, x_n^{\pm 1}]$, and fix $w \in \Gamma^n$. Show that for any $g \in \text{in}_w(I) \subseteq \mathbb{k}[x_1^{\pm 1}, \ldots, x_n^{\pm 1}]$ there is $f \in K[x_1^{\pm 1}, \ldots, x_n^{pm1}]$ with $\text{in}_w(f) = g$. (Hint: 2.4.2 of draft).

(2) Let $K = \overline{K}$. Show that if $V(I) = V(J)$ then $\text{trop}(V(I)) = \text{trop}(V(I))$ (ie that $\cap_{f \in I} \text{trop}(V(f)) = \cap_{g \in J} \text{trop}(V(g))$).

(3) Show that if $X = X_1 \cup X_2$ where X_1, X_2 are varieties in $(K^*)^n$, then $\text{trop}(X) = \text{trop}(X_1) \cup \text{trop}(X_2)$.

(4) Compute all initial ideals of of $I = \langle 7x_0^2 + 8x_0x_1 - x_1^2 + x_0x_2 + 3x_2^2 \rangle \subseteq \mathbb{C}[x_0, x_1, x_2]$, and draw the Gröbner complex of I. Repeat for the ideal $I = \langle tx_1^2 + 3x_1x_2 - tx_2^2 + 5x_0x_1 - x_0x_2 + 2tx_0^2 \rangle \subseteq \mathbb{C}\{\{t\}\}[x_0, x_1, x_2]$.

(5) Show that if K has the trivial valuation, then the Gröbner complex is a fan, so every polyhedron is a cone.