MATH 551 MIDTERM

October 19

Name: ______________________________

Numeric Student ID: ____________________

Instructions: Print your name in the space provided. During the test you may not use notes, books, or calculators. Read each question carefully. State clearly any theorems you use from class or the textbook, and make sure that their hypotheses are satisfied before you apply them. There are 5 questions. You have 80 minutes to do all the problems.

Question	Score	Maximum
1		10
2		10
3		10
4		10
5		10
Total		50

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(1) (a) Write down the definition of a product in a category.

(b) Recall that the category of pointed sets has objects a pair (S, x), where S is a set, and $x \in S$, and morphisms $f : (S, x) \to (T, y)$ functions f from S to T with $f(x) = y$. Show that products exist in this category, and describe them explicitly.

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(2) Let G be a finitely generated abelian group. Show that any subgroup H of a finitely generated abelian group is finitely generated and that if G has a generating set of size n, then H has a generating set of size n or smaller.

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(3) Let H be a subgroup of a group G, with $[G : H] = n < \infty$. Show that there is a normal subgroup K of G with $[G : K] \leq n!$. (Hint: Is there a relevant group action?).

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(4) Let H and N be subgroups of G, with N normal in G, and $G = NH$. Show that there is a homomorphism $\theta : H \to \mathrm{Aut}(N)$ for which G is isomorphic to $N \rtimes H$. (Hint: Think about conjugation).

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(5) If G is a group of order 2002, show that G has a cyclic subgroup of index 2.