MATH 551 HOMEWORK 9

DUE WEDNESDAY, NOVEMBER 16

You are encouraged to work on the homework together, but your final write-up should be your own. Please write down on your homework the name of any collaborators. No late homework will be accepted. "Hungerford I.1.3" means Question 3 in the exercises at the end of Section 1 of Chapter 1.

(1) Hungerford, IV.1.3
(2) Hungerford, IV.2.6
(3) Hungerford, IV.2.13
(4) A *projective module* P is an R module such that if $g : A \to B$ is a surjective map of R-modules, and $f : P \to B$ is a R-module homomorphism, then there is a R-module homomorphism $h : P \to A$ such that $f = gh$.
  (a) Show that every free R-module is projective.
  (b) Show that if $0 \to A \to B \to P \to 0$ is a short exact sequence of R-modules, and P is projective, then $B \cong A \oplus P$. Conclude that there is some free module F and R-module K for which $F \cong K \oplus P$ (ie P is a summand of a free module).
(5) Hungerford, IV.6.1
(6) **Spring 2004** In the ring $\mathbb{Q}[x]$ of polynomials in one variable with rational coefficients, all ideals are principal. This is no longer true for $\mathbb{Z}[x]$, but it still true that all ideals are finitely generated (this follows from the Hilbert basis theorem - see the commutative algebra class).
  (a) Exhibit an ideal of $\mathbb{Z}[x]$ that is not principal. Justify your answer.
  (b) Is there an upper bound on the minimum number of generators of an ideal in $\mathbb{Z}[x]$? Justify your answer.