MATH 551 HOMEWORK 11

DUE MONDAY, DECEMBER 12

Same caveats as for previous homeworks (deleted for space reasons!).

(1) Hungerford, IV.5.2
(2) Hungerford, IV.5.4
(3) Hungerford, IV.5.7
(4) Hungerford, IV.5.9
(5) If A is the matrix of a linear map ϕ from an n-dimensional vector space V over a field k to itself, and B is the matrix of a linear map ψ from an m-dimensional vector space W over k to itself, what is the matrix of $\phi \otimes \psi : V \otimes_k W \to V \otimes W$?
(6) Show that the map from the category of R-modules to itself that takes an R module A to $A \otimes_R B$, where B is a fixed R-module, and takes a morphism f to $f \otimes 1$ is a functor.
(7) Compute the Jordan and rational canonical forms of the following matrix:
$$\begin{pmatrix} 1 & 3 & 0 \\ -1 & 1 & 3 \\ 0 & 1 & 1 \end{pmatrix}.$$
(8) Hungerford VII.4.5
(9) Hungerford VII.4.13
(10) **Fall 2001** Suppose that V is a real vector space of finite dimension n and $T : V \to V$ is a linear transformation with no repeated eigenvalues. Show that there exists a vector $v \in V$ such that $\{v, Tv, T^2v, \ldots, T^{n-1}v\}$ is a basis of V.
(11) **Fall 2002** Suppose that A and B are $n \times n$ complex nilpotent matrices with the same rank and the same minimal polynomial.
   (a) If $n = 6$, prove that A and B are similar.
   (b) If $n = 7$, is this still true? Either prove or give a counterexample.
(12) **Spring 2004** Let $f(x) = (x-1)(x+1)^2$ and $g(x) = (x-1)(x+1)(x+2)$ in $\mathbb{Q}[x]$. find a 3×3 rational matrix A such that $g(A) = 0$, and the characteristic polynomial of A is $-f$. Here the characteristic polynomial is defined as $\det(A - xI)$.