MATH 428 HOMEWORK 9

DUE THURSDAY, 12/9/04

(1) Three people A, B, and C all apply for jobs at companies 1, 2, and 3. Each person ranks the three companies in order of preference, and each company ranks the people.
   (a) Give an example of such a ranking for which there is only one stable matching.
   (b) Give an example of such a ranking for which there are three stable matchings.

(2) In class we gave two (equivalent) algorithms to find a stable matching: "companies choose first" and "candidates choose first". Show that if these two algorithms give the same answer then this is the only stable matching in the case where there are three companies and three candidates. Extra credit: Do the general case.

(3) Wilson, Problem 26.2.

(4) We can extend the proof of Birkhoff's theorem given in class to show that if A is an $n \times n$ matrix with non-negative real entries with all rows and columns summing to one (called a *doubly stochastic matrix* for its probability interpretation), then $A = \sum_i \lambda_i P_i$, where P_i is a permutation matrix, and $0 < \lambda_i$, with $\sum_i \lambda_i = 1$. Illustrate this for the matrix

$$A = \begin{pmatrix} 2/5 & 1/5 & 2/5 \\ 1/5 & 4/5 & 0 \\ 2/5 & 0 & 3/5 \end{pmatrix}.$$

(5) Wilson, Problem 28.1.