# INTRODUCTION TO MACAULAY2

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The main source of information about Macaulay2 is the webpage: `http://www.math.uiuc.edu/Macaulay2/`.

Macaulay2 is installed on the Maths Linux machines on the ground floor. On the Macaulay2 webpage are files you can download to install it on your own machine. This is easier for Macs or machines running Linux than for machines running Windows. Another option is to use the web interface linked to from `http://www.math.uiuc.edu/Macaulay2/TryItOut/`.

To start Macaulay2 on a Unix system (Linux or MacOS), type `M2` at the command line.

You should see something like:

```
Macaulay2, version 1.9.2
with packages: ConwayPolynomials, Elimination, IntegralClosure, LLLBases,
               PrimaryDecomposition, ReesAlgebra, TangentCone

i1 :
```

The `i1 :` is the prompt.

The first thing we want to do is define a polynomial ring. Your main choice of fields is $\mathbb{Q}$ (written `QQ`) or a finite field $\mathbb{F}_p$ (written `ZZ/p`). The integers are represented by `ZZ`. Do not try to use the complex numbers as coefficients. To define a polynomial ring over $\mathbb{Q}$, we type:

```
i1 : R=QQ[a,b,c,d]
```

which produces:

```
o1 = R

o1 : PolynomialRing
```

The ring $R$ is now the polynomial ring in four variables with coefficients in $\mathbb{Q}$. The default term order is graded reverse lexicographic. If you wanted to use the lexicographic order, for example, you would have instead typed:

```
i2 : S=QQ[a,b,c,d,MonomialOrder=>Lex]
```

To switch back to $R$ type `use R`. To define an ideal in $R$, for example $I = \langle c^2 - bd, bc - ad, b^2 - ac \rangle$, type:

```
i3 : I=ideal(c^2-b*d, b*c-a*d, b^2-a*c)

             2                     2
o3 = ideal (c  - b*d, b*c - a*d, b  - a*c)

o3 : Ideal of R
```

Notice the way the exponents are on a separate line in the output. This can be annoying - one way to avoid it is to type:

```
o4 : toString I

o4 = ideal(c^2-b*d,b*c-a*d,b^2-a*c)
```

Another (more common) way to get around this problem by using it inside `emacs`. See the webpage for details.

To compute a Gröbner basis of the ideal type:

```
i5 : gb I

o5 = GroebnerBasis[status: done; S-pairs encountered up to degree 2]
```

To actually see the Gröbner basis you type:

```
i6: gens gb I

o6 = | c2-bd bc-ad b2-ac |

             1       3
o6 : Matrix R  <--- R
```

To get the initial ideal type:

```
i7 : leadTerm I

o7 = | c2 bc b2 |

             1       3
o7 : Matrix R  <--- R
```

The command `leadTerm` also finds the lead term of a polynomial.

To find the remainder when dividing a polynomial by a Gröbner basis type:

```
i8 : f=b*d-c^2

         2
o8 = - c  + b*d

o8 : R
```

```
i9 : f % I

o9 = 0

o9 : R

i10: g=b^3*c^3

       3 3
o10 = b c

o9 : R

i11 : g % I

       3 3
o11 = a d

o11 : R
```

The remainder on division (`%`) by $I$ is the remainder on division by the *reduced* Gröbner basis for $I$ with respect to the given term order.

You should now look at the documentation on the Macaulay2 webpage, particularly the section labelled *Getting started*. For help on a particular command, type `viewHelp commandName` (e.g., `viewHelp ring`).