244 SPRING 2006 SAMPLE MIDTERM 2 SOLUTIONS

SECTIONS 05, 06, 08

(1) The general solution is $y(t) = c_1 e^t \cos(2t) + c_2 e^t \sin(2t)$, so the solution to the initial value problem is $y(t) = -e^{t-\pi/2}\sin(2t)$.
(2) $y(t) = 2te^{3t}$.
(3) The general solution is $y(t) = 1/6t^3e^t + 4 + c_1e^t + c_2te^t$, so the solution is $-3e^t + 4te^t + 1/6t^3e^t + 4$.
(4) The general solution to the homogeneous equation is $y(t) = c_1e^{2t} + c_2e^{-t}$, so we look for a solution of the form $y(t) = v_1(t)e^{2t} + v_2(t)e^{-t}$. We differentiate and set $v_1'e^{2t} + v_2'e^{-t} = 0$. This leads to the other equation $2v_1'e^{2t} - v_2'e^{-t} = 2e^{-t}$. Solving these two equations for v_1', v_2' we get $v_1' = 2/3e^{-3t}$, and $v_2' = -2/3$, leading to $v_1 = -2/9e^{-3t}$ and $v_2 = -2/3t$. Thus $y = -2/9e^{-t} - 2/3te^{-t}$ is a solution to the equation. Since e^{-t} is a solution to the homogeneous equation we can remove that from the solution to get another solution being $y = -2/3te^{-t}$.
(5) $y(t) = c_1 + c_2e^t + c_3e^{-t} + \cos(t)$.
(6) $y(t) = c_1(|x-2|)^{-2}\sin(2\ln(|x-2|)+c_2(|x-2|)^{-2}\cos(2\ln(|x-2|)$.
(7) Let $x_1 = u$, and $x_2 = x_1'$. Then we have the system $x_1' = x_2, x_2' = 2\cos(3t) - 4x_1 - 0.25x_2$, with initial conditions $x_1(0) = 1$, $x_2(0) = -2$.
(8) (a) $A + B = \begin{pmatrix} 3+2t & t^2+e^{-t} \\ e^t+t+1 & 2t-1 \end{pmatrix}$.

(b) $AB = \begin{pmatrix} t^3+t^2+6t & 3\,e^{-t}+t^3 \\ 2\,te^t+t^2-1 & t^2-t+1 \end{pmatrix}$.

(c) $d/dt(AB) = \begin{pmatrix} 3t^2+2t+6 & -3\,e^{-t}+3t^2 \\ 2(t+1)e^t+2t & 2t-1 \end{pmatrix}$.
(9) $\det(A-\lambda I) = (5-\lambda)(1-\lambda)+3 = \lambda^2-6\lambda+8 = (\lambda-4)(\lambda-2)$. So the eigenvalues of A are $\lambda = 2$ and $\lambda = 4$. For $\lambda = 2$ the eigenvectors are all multiples of the vector $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$, while for $\lambda = 4$ the eigenvectors are all multiples of the vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.
(10) $W(\mathbf{x}^{(1)}, \mathbf{x}^{(2)})(t) = \begin{vmatrix} t & t^2 \\ 1 & 2t \end{vmatrix} = t^2$.