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(1) Find the solution of the initial value problem: $y'' - 2y' + 5y = 0$, $y(\pi/2) = 0, y'(\pi/2) = 2$.

(2) Solve the initial value problem: $y'' - 6y' + 9y = 0$, $y(0) = 0$, $y'(0) = 2$.

(3) Find the solution of the initial value problem: $y'' + 2y' + y = te^t + 4$, $y(0) = 1$, $y'(0) = 1$.

(4) Solve the following differential equation using the method of variation of parameters: $y'' - y' - 2y = 2e^{-t}$.

(5) Determine the general solution of the differential equation: $y''' - y' = 2\sin(t)$.

(6) Give the general solution to $(x-2)^2 y'' + 5(x-2)y' + 8y = 0$.

(7) Transform the following initial value problem into an initial value problem for two first order equations: $u'' + 0.25u + 4u = 2\cos(3t)$, $u(0) = 1, u'(0) = -2$.

(8) If $A = \begin{pmatrix} 3 & t^2 \\ e^t & t-1 \end{pmatrix}$ and $B = \begin{pmatrix} 2t & e^{-t} \\ t+1 & t \end{pmatrix}$, compute:
   (a) $A + B$
   (b) AB
   (c) $d/dt(AB)$

(9) Find all eigenvalues and eigenvectors of $A = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}$.

(10) Compute the Wronskian of the vectors $\mathbf{x}^{(1)} = \begin{pmatrix} t \\ 1 \end{pmatrix}$ and $\mathbf{x}^{(2)} = \begin{pmatrix} t^2 \\ 2t \end{pmatrix}$.