MATH 244 SAMPLE FINAL

SECTION 05, 06, 08, SPRING 2006

Caveats: This is closer in length to the actual final than the sample midterms have been. As a result it does not cover absolutely everything we have done in the class (so there could be a question on the exam not exactly like these questions). Just doing these questions is not sufficient to prepare yourself for the final. You should also do more questions from the book (do the ones in the same blocks as other homework questions).

(1) Find the solution of the initial value problem: $y' - y = 2te^{2t}$, $y(0) = 1$.

(2) Solve the following initial value problem: $y' = (1 - 2x)/y$, $y(1) = -2$.

(3) A tank originally contains 100 gallons of fresh water. Then water containing 1/2 pound of salt per gallon is poured into the tank at a rate of 2 gallons per minute, and the mixture is allowed to leave at the same rate. After 10 minutes the proeces is stopped, and fresh water is poured into the tank at a rate of 2 gallons per minute with the mixture again leaving at the same rate. Find the amount of salt in the tank at the end of an additional 10 minutes.

(4) Use the backwards Euler's method to estimate $y(0.1)$ for the solution to the initial value problem $y' = 2y - 3t$, $y(0) = 1$.

(5) Find a fundamental set of solutions to the equation $y'' - 2y' + 6y = 0$.

(6) Solve the differential equation $y'' - 4y' + 4y = e^{2t}$, $y(0) = 1$, $y'(0) = 0$.

(7) Find the general solution of the equation $y''' - y' = 2\sin(t)$.

(8) (a) Solve the differential equation $x^2y'' - xy' + y = 0$.
   (b) Find all values of α for which all solutions of $x^2y'' + \alpha xy' + (5/2)y = 0$ approach zero as $x \to 0$.

(9) Solve the initial value problem:

$$\begin{aligned} x' &= -2x + y \\ y' &= -5x + 4y \end{aligned}$$

where $x(0) = 1, y(0) = 3$.

(10) Find the general solution to the system:

$$\mathbf{x}' = \begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}.$$

(11) Find the general solution to the system:

$$\mathbf{x}' = \begin{pmatrix} 4 & -2 \\ 8 & -4 \end{pmatrix} \mathbf{x}.$$

(12) Sketch some trajectories of the following equations:

(a)
$$\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} \mathbf{x},$$

(b)
$$\mathbf{x}' = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} \mathbf{x},$$

(c)
$$\mathbf{x}' = \begin{pmatrix} 5 & -6 \\ 4 & -5 \end{pmatrix} \mathbf{x}.$$

(13) Consider the system:

$$\begin{aligned} \frac{dx}{dt} &= 1 - xy \\ \frac{dy}{dt} &= x - y^3 \end{aligned}$$

(a) Calculate the critical points of this system.
(b) For each critical point, determine whether it is asymptotically stable, stable, or unstable.

(14) Solve the differential equation $y'' - xy' - y = 0$ using series solution about $x_0 = 0$.