MA 243 HOMEWORK 5

DUE: THURSDAY, 8 NOVEMBER, 2007, BY 12PM

Hand in the problems in Section B *only* to the boxes outside the undergraduate office. You are encouraged to work together on the problems, but your written work should be your own.

A : Warm-up problems

(1) Use the spherical cosine law to compute the distance between the points $(1, 0, 0)$ and $(0, 1/\sqrt{2}, 1/\sqrt{2})$ on S^2.
(2) Describe all motions of $\mathbb{E}^2$ you can obtain by repeatedly reflecting in the y axis and reflecting in the line $x = 1$.

B: Exercises

(1) Consider the motions of $\mathbb{E}^2$ given by the reflection T in the x-axis and the reflection S in the y-axis. How many different motions of $\mathbb{E}^2$ can you obtain by repeated composition of T and S? (for example, $T \circ S$, $T \circ S \circ T \circ S \circ S$).

    How does your answer change if S changes to the reflection in the line $y = -\sqrt{3}x$?
(2) Use the main formula of spherical trig to calculate the distance from London to Christchurch, NZ on the surface of the earth, using that London is approximately 51° North, and Christchurch is approximately 43° South, 172° East. Recall that latitude is measured from the equator 0° north to the North Pole = 90° N, and longitude is measured from the Greenwich observatory, which is in London. The circumference of the earth is 40,000 km by the definition of kilometer.
(3) (Notes, Exercise 3.5). Let α, β, γ be the side lengths of a spherical triangle ΔPQR and a, b, c be the opposite spherical angles. use the spherical cosine law to prove that $|\beta - \gamma| \leq \alpha \leq \beta + \gamma$ and $\alpha + \beta + \gamma \leq 2\pi$.

    Note that there is a hint/partial solution for this exercise. Your answer must contain much more detail!
(4) Prove the lemma we stated in class: P, Q, R are collinear if and only if either $d(P, Q) + d(Q, R) = d(P, R)$ after relabelling, or $d(P, Q) + d(Q, R) + d(P, R) = 2\pi$.

C: Extensions

(1) Consider the motions of $\mathbb{E}^2$ given by the reflection T in the x-axis and the reflection S in the line $y = \tan(2\pi/n)$ for a fixed $n \geq 3$. How many different motions of $\mathbb{E}^2$ can you obtain by repeated composition of T and S? How does your answer change if S is a general line $y = cx$ for a fixed c not equal to $\tan(\pi/n)$ for some n? What if the two lines of reflection do not intersect?