MA 243 HOMEWORK 6

DUE: THURSDAY, 20, 2008, BY 12PM

Hand in the problems in Section B *only* to the boxes outside the undergraduate office. You are encouraged to work together on the problems, but your written work should be your own.

A : Warm-up problems

(1) Calculate the intersection (if any) of the following lines in $\mathbb{H}^2$, and any common perpendiculars:
   (a) $L = \{x = y\} \cap \mathbb{H}^2, L' = \{t = 2x\} \cap \mathbb{H}^2$
   (b) $L = \{x = 2t\} \cap \mathbb{H}^2, L' = \{y = 3t\} \cap \mathbb{H}^2$
   (c) $L = \{5x = 4t\} \cap \mathbb{H}^2, L' = \{5y = 3t\} \cap \mathbb{H}^2$

(2) Consider the line $L = \{y = 0\} \cap \mathbb{H}^2$ in $\mathbb{H}^2$. Show that this corresponds to the line $y = 0$ in the Poincaré disk model. (See B3 for the extension).

B: Exercises

(1) Show that if $L = \Pi \cap \mathbb{H}^2$ is a line in $\mathbb{H}^2$ then there are an infinite number of vectors $\mathbf{v} \in \Pi$ with $q_L(\mathbf{v}) = 1$. Deduce that given a line L and a point P not on L there are an infinite number of lines L' passing through P and not intersecting L. Compare with $\mathbb{E}^2$ and S^2.

(2) (a) Show that if $T(\mathbf{x}) = A\mathbf{x}$ is a hyperbolic motion then $\det(A) = \pm 1$ (thus hyperbolic motions are either orientation preserving or reversing!)
   (b) Conclude that either 1 or -1 is an eigenvalue of A. (See below for a continuation of this question).

(3) Consider the line $L = \{t = 2x\} \cap \mathbb{H}^2$ in $\mathbb{H}^2$. Show that in the Poincaré disk model of $\mathbb{H}^2$, L is taken to an arc of the circle of radius $\sqrt{(3)}$ centred at the point $(2, 0)$ (using the identification of projecting from $(-1, 0, 0)$ as in the diagram on p70 of Cannon, Floyd, Kenyon and Parry, which is linked on the main webpage under announcements).

C: Extensions

(1) (Hyperbolic motions continued). Show that if $\mathbf{v}$ is an eigenvector of A with eigenvalue ± 1, then we can find an appropriate change of basis so that A is in block form with one block of size one and one block of size two. What must the block of size two look like? What does this say about the classification of hyperbolic motions?

(2) In class we used the hyperbolic cosine law to prove the triangle inequality, and then argued from the triangle inequality that collinearity is preserved by distance, so motions must take lines to lines. However the first step in the proof of the hyperbolic cosine law was to choose a good coordinate system in which $P = (1, 0, 0)$ and $Q = (\cosh(\beta), \sinh(\beta), 0)$. How do we know that that is possible in such a way that the line joining P to Q is taken to the line joining $(1, 0, 0)$ to $(\cosh(\beta), \sinh(\beta), 0)$? In other words, why does the map taking P to $(1, 0, 0)$ and Q to $(\cosh(\beta), \sinh(\beta), 0$ take lines to lines?

(3) (Highly recommended!!!) There are many many interesting problems in the notes/book we did not have time to cover. For example, prove the spherical or hyperbolic sine law. What is the circumference of a spherical or hyperbolic circle? The area?