MA 243 HOMEWORK 2

DUE: THURSDAY, OCTOBER 16, 2008, BY 12PM

Hand in the problems in Section B *only* to the boxes outside the undergraduate office. You are encouraged to work together on the problems, but your written work should be your own.

A : Warm-up problems

(1) Which of the following sets of three points are collinear?
   (a) $\{(1,0,0),(1,2,3),(1,4,6)\}$,
   (b) $\{(1,1,1),(1,1,3),(1,5,4)\}$,
   (c) $\{(1,1,1),(1,1,3),(1,1,4)\}$,

(2) Fix two points P and Q of $\mathbb{E}^2$ and describe in coordinates the motion given by first rotating by an angle of $\pi/4$ about P, and then reflecting in the line $\overline{PQ}$.

(3) Show that the composition of two motions is a motion.

B: Exercises

(1) Let $T : \mathbb{E}^n \to \mathbb{E}^n$ be a motion. Fix a choice of coordinates $\phi : \mathbb{E}^n \to \mathbb{R}^n$.
   (a) Show that $\phi \circ T : \mathbb{E}^n \to \mathbb{R}^n$ is another choice of coordinates.
   (b) Define $T' : \mathbb{R}^n \to \mathbb{R}^n$ by $T' = \phi \circ T \circ \phi^{-1}$. Show that T' is a distance preserving map (so $|T'(\mathbf{x}) - T'(\mathbf{y})| = |\mathbf{x} - \mathbf{y}|$).
   (c) Show that if $\psi : \mathbb{E}^n \to \mathbb{R}^n$ is another choice of coordinates, then $T : \mathbb{E}^n \to \mathbb{E}^n$ defined by $T = \phi^{-1} \circ \psi$ is a motion.

(2) Let T be the motion obtained by rotating by θ anti-clockwise about a point P in $\mathbb{E}^2$, and let S be the motion obtained by rotating by ω anti-clockwise about the same point P. Write down the matrices A and B for T and S in some choice of coordinates. Describe the motion $T \circ S$ geometrically, and write down its matrix in the same choice of coordinates. Compare this to the matrix AB (unsimplified) and explain your answer.

(3) Let T be the motion of $\mathbb{E}^2$ of anti-clockwise rotation by $\pi/2$ about a point P, and let S be the motion of $\mathbb{E}^2$ of anti-clockwise rotation by $\pi/2$ about a point Q distance one from P. Fix a coordinate choice in which P is taken to $(0,0)$, and Q is taken to $(1,0)$.

(a) Write down the expression for T in these coordinates.
(b) Write down the expression for S in *these* coordinates. (Hint: You may want to choose a more convenient coordinate choice and then rewrite in these coordinates).
(c) Write down the composition $S \circ T$ in these coordinates
(d) Describe $S \circ T$ geometrically.

C: Extensions

(1) In $\mathbb{E}^2$ we know some important motions are translation, rotation, and reflection. What other motions can you obtain by composing these motions (eg a translation followed by a rotation)? We will discuss this in class over the next weeks.