MA 243 HOMEWORK 1

SOLUTIONS

B: Exercises

(1) **Who was Lobachevsky? Why is he relevant to this module? Cite any source you consult. Internet sources are ok for this question, though often not for scholarly work. Write a short paragraph (approximately five to ten sentences) summarizing what you discover about him.**

A good source for mathematical biography is the page at St Andrews:
`http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Lobachevsky`
Warning: Google might have led you to Tom Lehrer. While he is entertaining (and a mathematician as well!), the Lobachevsky song has nothing to do with the historical figure. Beware of internet research!

(2) **Describe the line joining $(1, 3, 0)$ and $(-1, 2, 3)$ in $\mathbb{R}^3$ both parametrically, and by equations. (Hint: You may need more than one equation).**

The line is $\{(1, 3, 0)+\lambda(-2, -1, 3) : \lambda \in \mathbb{R}\}$. This is $\{(x, y, z) : x - 2y = -5, 3y + z = 9\}$.

(3) **What can you conclude if $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^n$ with $\mathbf{x} \in [\mathbf{y}, \mathbf{z}]$, $\mathbf{y} \in [\mathbf{x}, \mathbf{z}]$, and $\mathbf{z} \in [\mathbf{x}, \mathbf{y}]$? Justify your answer.**

Note: $\mathbf{x} \in [\mathbf{y}, \mathbf{z}]$ if $\mathbf{x} = \mathbf{y} + \lambda(\mathbf{y} - \mathbf{z})$ for some $0 \leq \lambda \leq 1$.

If $\mathbf{x} \in [\mathbf{y}, \mathbf{z}]$, then $\mathbf{x} = \mathbf{y} + \lambda(\mathbf{z} - \mathbf{y})$ for some $0 \leq \lambda \leq 1$, so $|\mathbf{x} - \mathbf{y}| + |\mathbf{x} - \mathbf{z}| = |\mathbf{y} - \mathbf{z}|$. Similarly, $\mathbf{y} \in [\mathbf{x}, \mathbf{z}]$ implies that $|\mathbf{y} - \mathbf{z}| + |\mathbf{y} - \mathbf{x}| = |\mathbf{x} - \mathbf{z}|$. Substituting the second equation into the first we get $|\mathbf{x} - \mathbf{y}| = 0$, so $\mathbf{x} = \mathbf{y}$.

The same calculation applied to $\mathbf{x} \in [\mathbf{y}, \mathbf{z}]$ and $\mathbf{z} \in [\mathbf{x}, \mathbf{y}]$ gives $\mathbf{x} = \mathbf{z}$, so we conclude that $\mathbf{x} = \mathbf{y} = \mathbf{z}$.

(4) **Find linear transformations $T : \mathbb{R}^2 \to \mathbb{R}^2$ that takes the following ordered pairs to each other. In each case, describe T by giving a 2×2 matrix A with $T(\mathbf{x}) = A\mathbf{x}$.**

(a) $\{(1,0),(0,1)\}$, $\{(2,1),(5,3)\}$:

$$A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}.$$

(b) $\{(1,0),(0,1)\}$, $\{(3,5),(2,1)\}$:

$$A = \begin{pmatrix} 5 & 2 \\ 3 & 1 \end{pmatrix}.$$

(c) $\{(2,1),(5,3)\}$, $\{(1,0),(0,1)\}$:

$$A = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}.$$