MA 243 HOMEWORK 1

DUE: THURSDAY, OCTOBER 9, 2008, BY 12PM

Hand in the problems in Section B *only* to the boxes outside the undergraduate office. You are encouraged to work together on the problems, but your written work should be your own. Note that this first assignment is somewhat unusual in the questions A1 and B1.

A : Warm-up problems

(1) Show that the set $\{(x, y) \in \mathbb{R}^2 : 3x + 4y = 3\}$ is a line by giving the vector $\mathbf{u}, \mathbf{v}$ from lecture.
(2) Let $\mathbf{u} = (1, 0)$ and $\mathbf{v} = (2, 3)$. Write an equation for the line $\{\mathbf{u} + \lambda\mathbf{v} : \lambda \in \mathbb{R}\}$.
(3) Describe the line joining the points $(1, 2)$ and $(2, 1)$ both parametrically, and by equations.
(4) Explain how a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ is described by a matrix: $T(\mathbf{x}) = A\mathbf{x}$. How can you determine A?

B: Exercises

(1) Who was Lobachevsky? Why is he relevant to this module? Cite any source you consult. Internet sources are ok for this question, though often not for scholarly work. Write a short paragraph (approximately five to ten sentences) summarizing what you discover about him.
(2) Describe the line joining $(1, 3, 0)$ and $(-1, 2, 3)$ in $\mathbb{R}^3$ both parametrically, and by equations. (Hint: You may need more than one equation).
(3) What can you conclude if $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^n$ with $\mathbf{x} \in [\mathbf{y}, \mathbf{z}]$, $\mathbf{y} \in [\mathbf{x}, \mathbf{z}]$, and $\mathbf{z} \in [\mathbf{x}, \mathbf{y}]$? Justify your answer *carefully*.
(4) Find linear transformations $T : \mathbb{R}^2 \to \mathbb{R}^2$ that takes the following ordered pairs to each other. In each case, describe T by giving a 2×2 matrix A with $T(\mathbf{x}) = A\mathbf{x}$.
   (a) $\{(1, 0), (0, 1)\}$, $\{(2, 1), (5, 3)\}$;
   (b) $\{(1, 0), (0, 1)\}$, $\{(3, 5), (2, 1)\}$;
   (c) $\{(2, 1), (5, 3)\}$, $\{(1, 0), (0, 1)\}$.

C: Extensions

(1) If we want to describe a line in $\mathbb{R}^n$ by equations, how many equations do we need? What do they look like? (for example, what degree are they?) What does this have to do with your linear algebra module?

(2) Our distance function $d(\mathbf{x}, \mathbf{y}) = |\mathbf{x} - \mathbf{y}|$ comes from the *norm* $|\mathbf{x}| = \sqrt{\mathbf{x} \cdot \mathbf{x}} = \sqrt{\sum_{i=1}^n x_i^2}$. Two other norms on $\mathbb{R}^n$ are given by $|\mathbf{x}|_1 = \sum_{i=1}^n |x_i|$, and $|\mathbf{x}|_\infty = \max_{i=1}^n |x_i|$. We can then define $d_1(\mathbf{x}, \mathbf{y}) = |\mathbf{x} - \mathbf{y}|_1$ and $d_\infty(\mathbf{x}, \mathbf{y}) = |\mathbf{x} - \mathbf{y}|_\infty$.

(a) Calculate $d_1(\mathbf{x}, \mathbf{y})$ and $d_\infty(\mathbf{x}, \mathbf{y})$ for $\mathbf{x} = (1, 0, 0)$ and $\mathbf{y} = (-1, 2, 3)$.

(b) The metric d_1 is often called the "Manhattan metric". Can you see why?