Please let me know if any of the problems are unclear, have typos, or have any other mistakes. For any problem where n is general, feel free to restrict to the cases where n is at most three. (Or four, if you are a medium-dimensional topologist instead of a low-dimensional one.)

For this exercise sheet we will use the following notations: S^2 for the two-sphere, D^2 for the disk, P^2 for the real projective plane, A^2 for the annulus, M^2 for the Möbius band, T^2 for the two-torus, and K^2 for the Klein bottle.

Exercise 2.1.

1. Sketch pseudo-simplicial complexes representing A^2 and M^2.
2. Give proofs, directly from the definition, that A^2 is orientable and M^2 is not.
3. [Medium] Prove that orientability (of a combinatorial n–manifold) is preserved by Pachner moves.

Exercise 2.2. Suppose that X and Y are surfaces with boundary and $\phi\colon \partial X \to \partial Y$ is an isomorphism. We write $X \cup_\phi Y$ for the surface obtained by gluing X to Y via ϕ. Prove the following homeomorphisms hold; in each case you will need to find the correct gluing ϕ.

1. $S^2 \cong D^2 \cup_\phi D^2$
2. $P^2 \cong M^2 \cup_\phi D^2$
3. $T^2 \cong A^2 \cup_\phi A^2$
4. $K^2 \cong A^2 \cup_\phi A^2$
5. $K^2 \cong M^2 \cup_\phi M^2$

Exercise 2.3. Suppose that X, Y, and Z are connected surfaces. Recall that # is the connect sum operation. Prove the following homeomorphisms hold.

1. $X \cong X \# S^2$
2. $X \# Y \cong Y \# X$
3. $(X \# Y) \# Z \cong X \# (Y \# Z)$

If X and Y are also finite, then prove that $\chi(X \# Y) = \chi(X) + \chi(Y) - 2$.

Exercise 2.4. Prove the following homeomorphisms hold.

1. $A^2 \cong D^2 \# D^2$
2. $M^2 \cong P^2 \# D^2$

3. $K^2 \cong P^2 \# P^2$
4. $T^2 \# P^2 \cong K^2 \# P^2$

Exercise 2.5. Using the classification of surfaces (or otherwise) identify the surface in Figure 2.6.

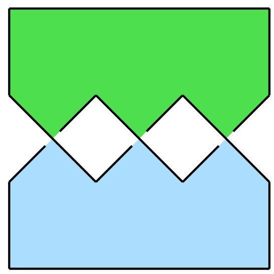

Figure 2.6: A Seifert surface for the trefoil knot