Please let me know if any of the problems are unclear, have typos, or have any other mistakes. For any problem where n is general, feel free to restrict to the cases where n is at most three. (Or four, if you are a medium-dimensional topologist instead of a low-dimensional one.)

Exercise 1.1. Using scissors and tape (available at the front of the room), make a Möbius strip. How many boundary components does it have? How many "sides"? What surface do you get if you cut the strip along its centre line? What do you get if you cut along a line one-third of the way in?

Now, instead of making a standard Möbius strip, make a strip with two or three half-twists, instead of one. Is this homeomorphic to the standard Möbius strip? If so, is it isototopic to the standard Möbius strip? What do you get if you cut it along its centre line?

Exercise 1.2. Let Δ^n be the standard n–simplex.

1. For each non-negative integer $k \leq n$ count the number of k–facets of Δ^n.
2. Using this or otherwise, compute the Euler characteristics of Δ^n and of $\partial\Delta^n$.
3. [Hard] More generally, suppose that C is a n–dimensional subcomplex of $\partial\Delta^{n+1}$. Compute the Euler characteristic of C.

Exercise 1.3. [Very hard] The *topologist's dunce cap* is the pseudo-simplical two-complex D consisting of a single triangle with all of its edges identified, as shown in Figure 1.4. Draw a good picture of D embedded in $\mathbb{R}^3$.

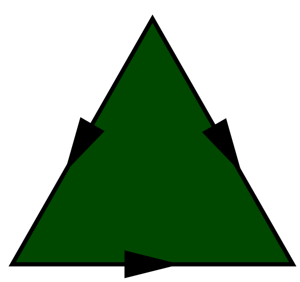

Figure 1.4: Glue the edges as shown to obtain the dunce cap.

Exercise 1.5. Let $X = \partial\Delta^3$ be the boundary of the tetrahedron. Let Y be the boundary of the octahedron. Find a sequence of Pachner moves that transforms X into Y. [Medium] What about getting to Z, the boundary of the icosahedron?

Exercise 1.6. Suppose that X is a pseudo-simplical n–complex. Suppose that v is a vertex of X.

1. Show that $\mathrm{link}_X(v)$ is a pseudo-simplical $(n-1)$–complex.
2. Suppose that $X \overset{P}{\to} Y$ is a Pachner move that does not destroy v. Prove that $\mathrm{link}_X(v)$ and $\mathrm{link}_Y(v)$ are PL homeomorphic (that is, differ by a sequence of Pachner moves).

Exercise 1.7. Suppose that X is pseudo-simplicial n–complex and that $X \overset{P}{\to} Y$ is a single Pachner move.

1. Prove that X and Y have the same Euler characteristic.
2. Prove that if X is a combinatorial manifold then so is Y.

Remark 1.8. For manifolds with boundary, we must modify the definition of equivalence. Suppose that X is a combinatorial n–manifold with boundary. Suppose that Δ is an n–simplex. Suppose that $A \subset \partial X$ and $B \subset \partial\Delta$ are combinatorial $(n-1)$–balls that are isomorphic via $\phi\colon A \to B$. Then there is an *expansion* of X across A which gives $Y = X \cup_\phi \Delta$. A *collapse* is the opposite of an expansion.

We say that a pair of combinatorial n–manifolds with boundary are X and Y are *PL homeomorphic* if they are connected by a sequence of Pachner moves, collapses, and expansions.

Exercise 1.9. [Medium] Suppose that C is a n–dimensional subcomplex of $\partial\Delta^{n+1}$. Prove that C is empty, is all of $\partial\Delta^{n+1}$, or is a combinatorial n–ball.

Exercise 1.10. Give the complete list of connected combinatorial one-manifolds (possibly with boundary). Using this or otherwise, prove that if X is a finite combinatorial one-manifold, then the number of points of ∂X is even.