Fleischner's Theorem for Infinite Graphs

Angelos Georgakopoulos

Methematisches Seminar der Universität Hamburg

Fleischner's Theorem

Theorem (Fleischner '74)

The square of a finite 2-connected graph has a Hamilton cycle

Fleischner's Theorem

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The square of a finite 2-connected graph has a Hamilton cycle

Theorem (Thomassen '78)

The square of a locally finite 2-connected 2-indivisible graph has a spanning double ray

Double Rays

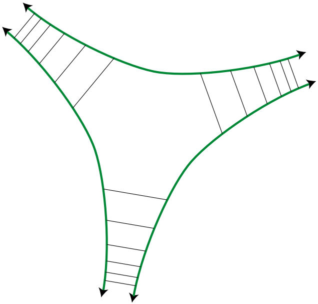
Theorem (Tutte '56)

Every finite 4-connected planar graph has a Hamilton cycle

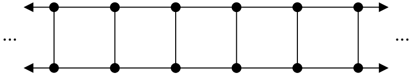
Theorem (Yu '05)

Every locally finite 4-connected planar 3-indivisible graph has a spanning double ray

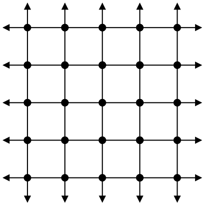
Infinite Cycles



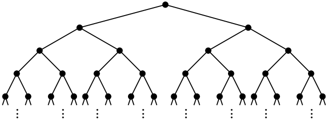
Ends



two ends



one end

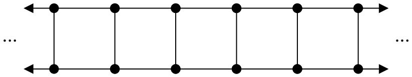


2^{ω} many ends

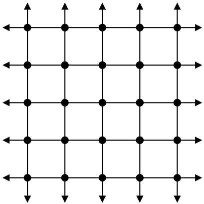
Ends

end: equivalence class of rays

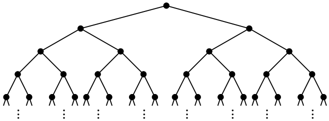
two rays are equivalent if no finite vertex set separates them



two ends

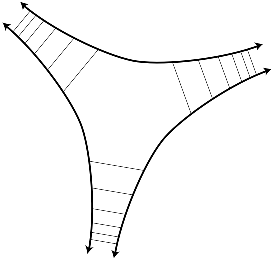


one end

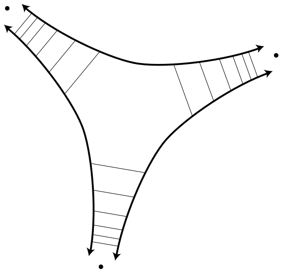


2^ω many ends

The Freudenthal Compactification



The Freudenthal Compactification



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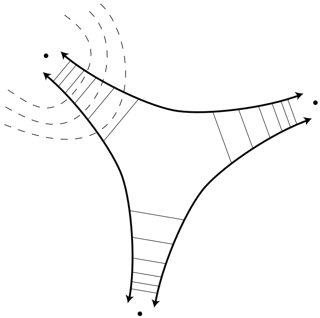
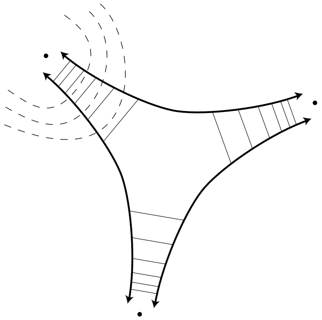


The Freudenthal Compactification



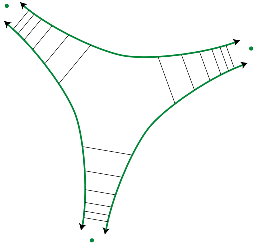
Every ray converges to its end!

Infinite Cycles

Circle (Diestel & Kühn):
A homeomorphic image of S^1 in the Freudenthal Compactification $|G|$ of G.

Infinite Cycles

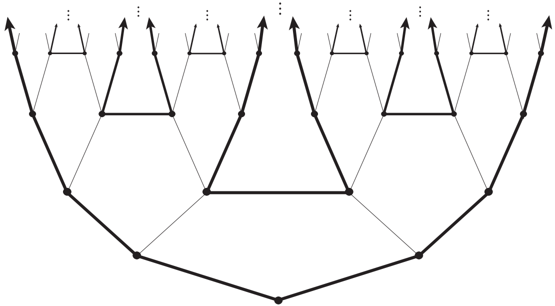
Circle:
A homeomorphic image of S^1 in $|G|$.



Hamilton circle:
a circle containing all vertices

Infinite Cycles

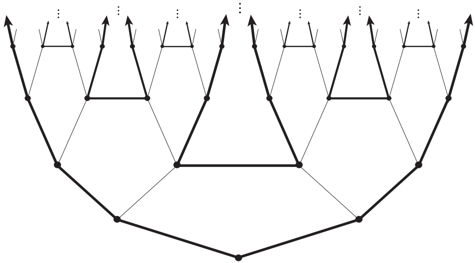
Circle:
A homeomorphic image of S^1 in $|G|$.



The Theorem

Theorem (G '06)

The square of a locally finite 2-connected graph has a Hamilton circle



Structure of the Proof

- make G eulerian by deleting edges and by doubling existing edges
- pick an Euler tour
- bridge crossings to turn the Euler tour into a Hamilton cycle

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Extra problems for infinite graphs:

- How do you define an Euler tour?

Euler Tours

Euler tour (Diestel & Kühn):
A continuous image of S^1 in $|G|$ traversing each edge exactly once.

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Extra problems for infinite graphs:

- How do you define an Euler tour?
- deleting edges may change the topology

Structure of the Proof

- make G eulerian by deleting edges and by doubling existing edges
- pick an Euler tour
- bridge crossings to turn the Euler tour into a Hamilton cycle

It will not work if we have too many crossings

Extra problems for infinite graphs:

- How do you define an Euler tour?
- deleting edges may change the topology
- need a special Euler tour

End-faithful Euler Tours

Theorem (G '06)

If a locally finite graph has an Euler tour then it also has one visiting each end exactly once.