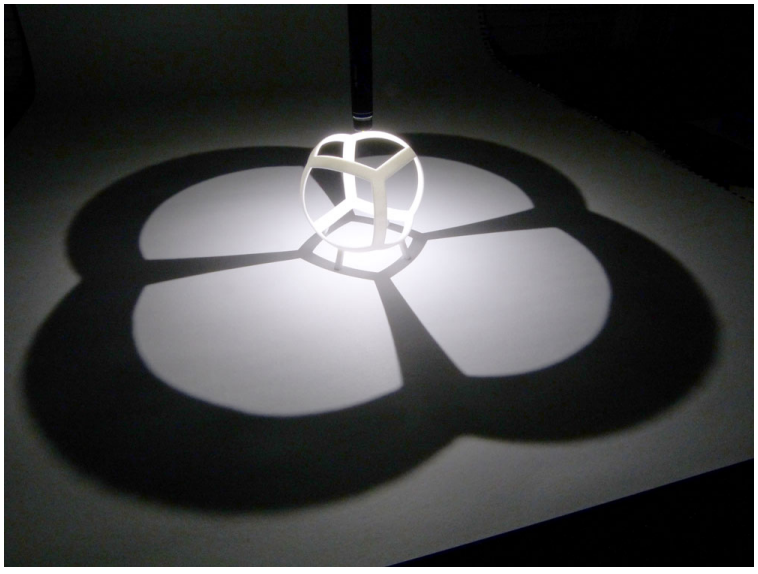
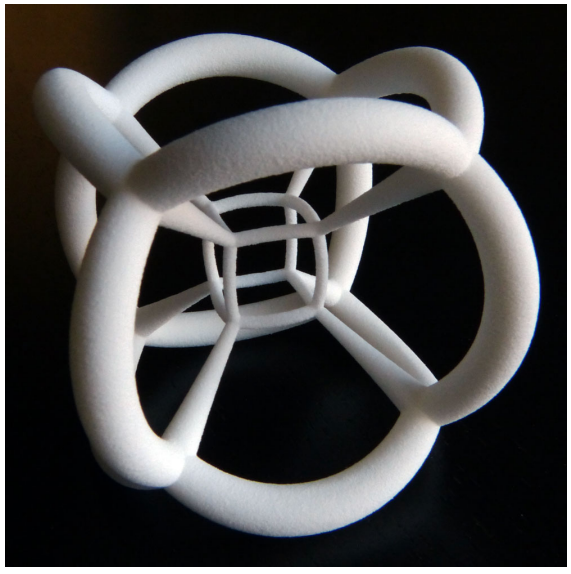
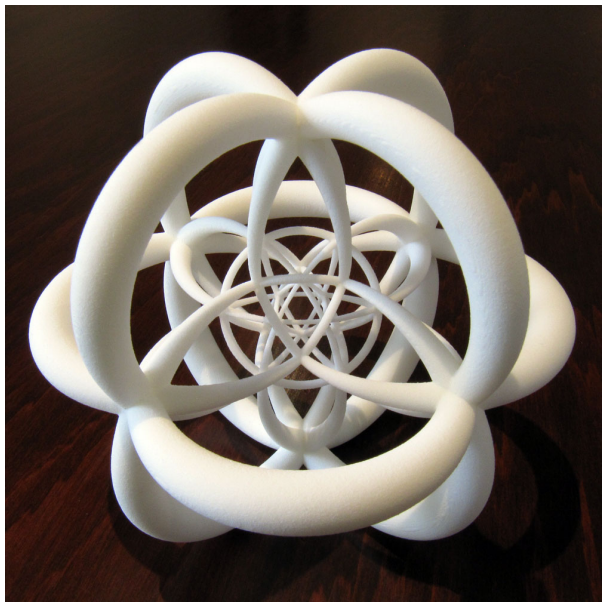



Henry Segerman
Oklahoma State University

Sculpture in 4-dimensions




3D printing technologies

Fused deposition modelling

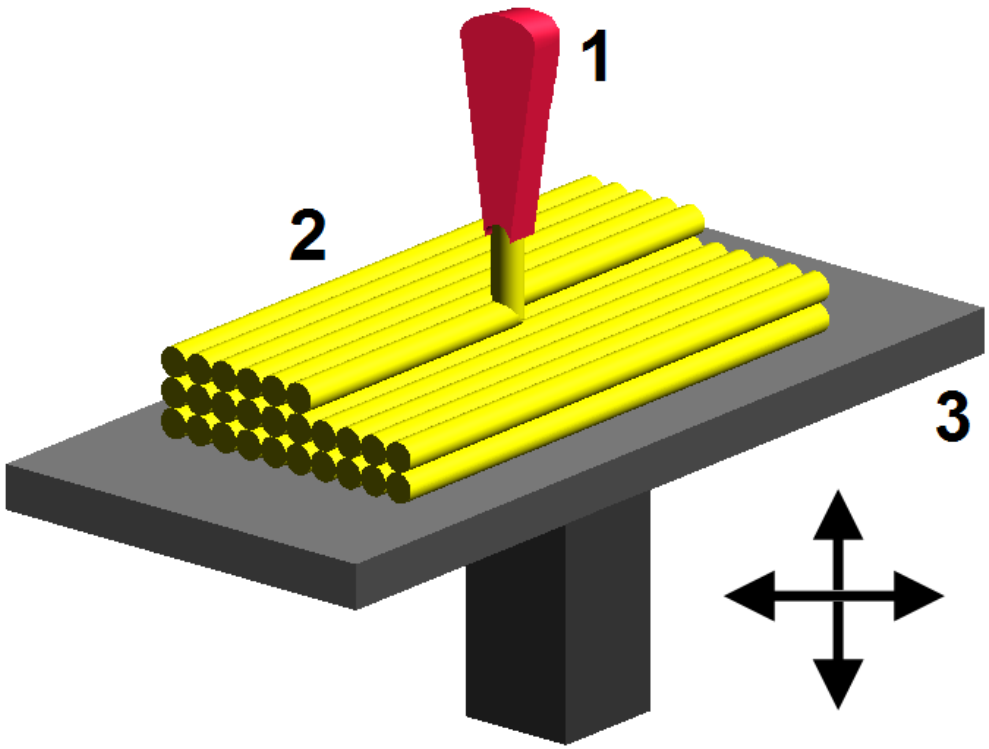


Image credit: Wikipedia - Zureks

3D printing technologies

Selective laser melting

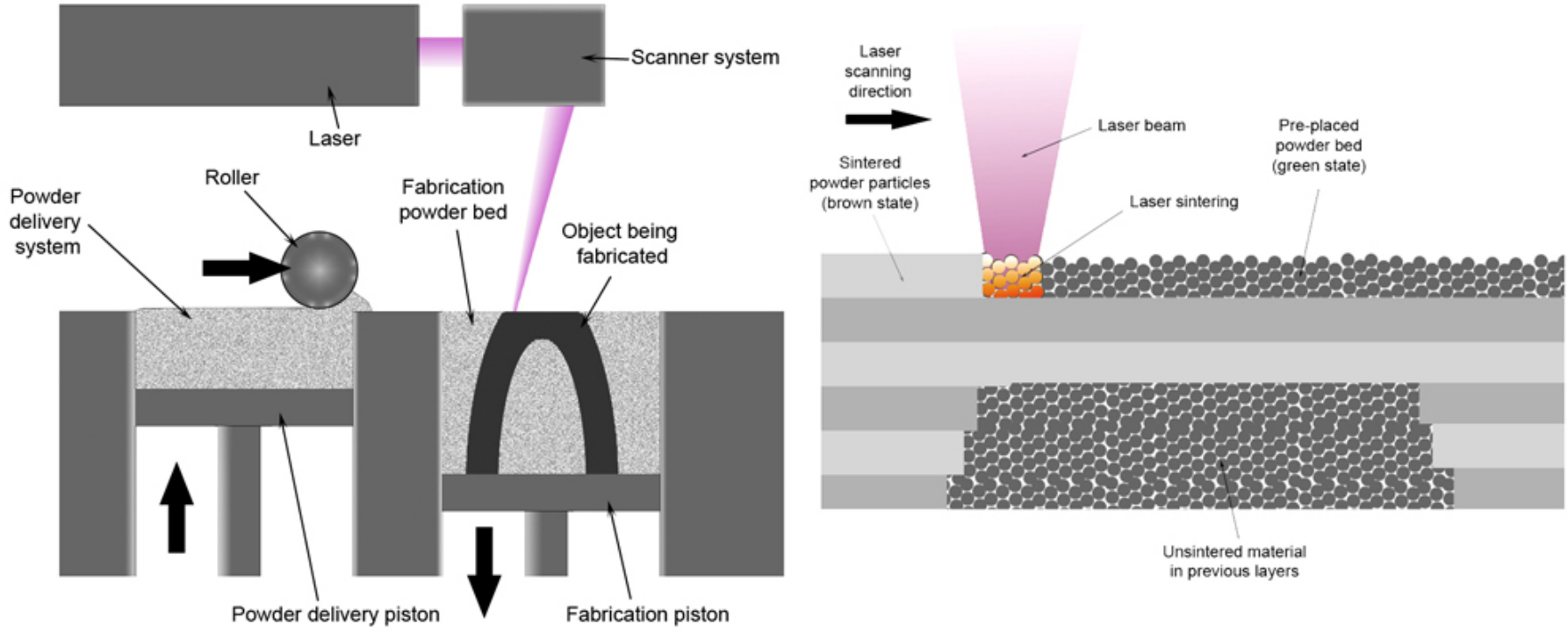
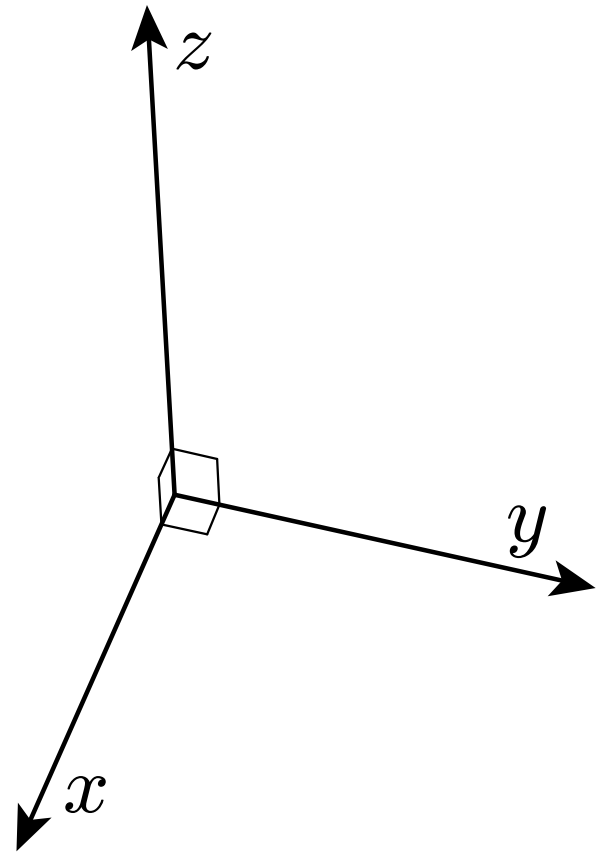


Image credit: Wikipedia - Materialgeeza

What is 4–dimensional space?

What is 4–dimensional space?

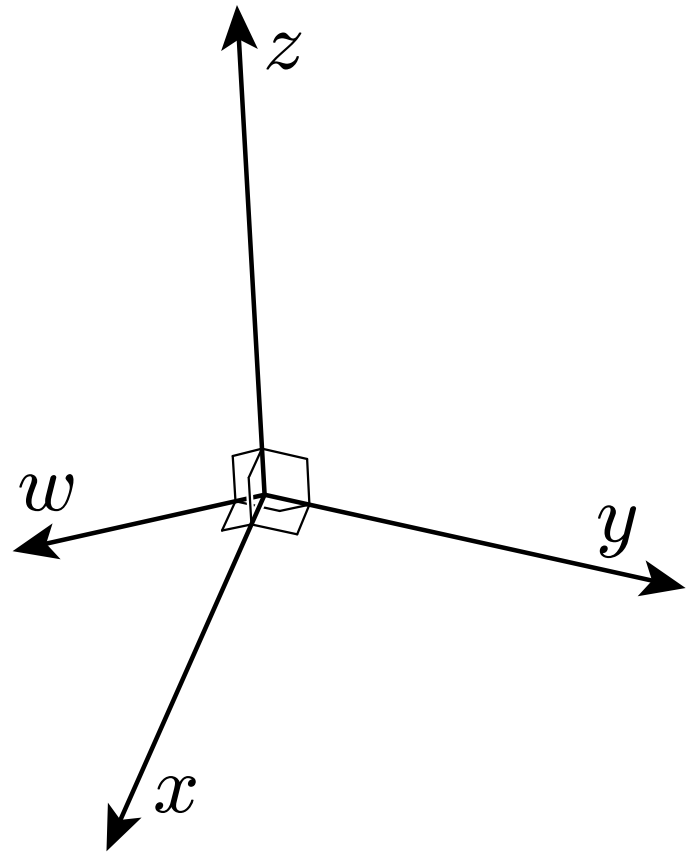
We describe a point in 3-dimensional space using three numbers, say (x, y, z).



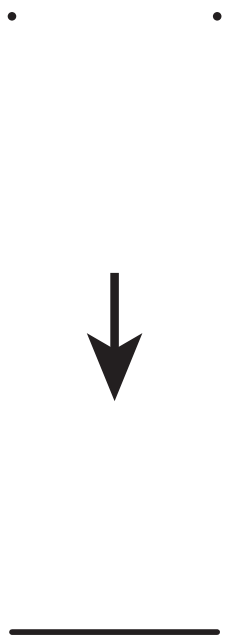
What is 4–dimensional space?

We describe a point in 3-dimensional space using three numbers, say (x, y, z).

A point in 4-dimensional space is given by four numbers, say (w, x, y, z).



Example: how to make a hypercube



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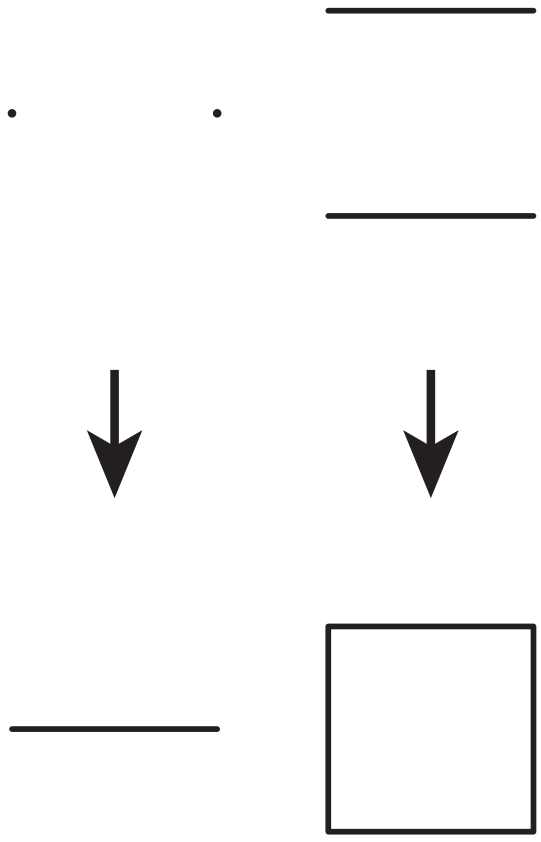

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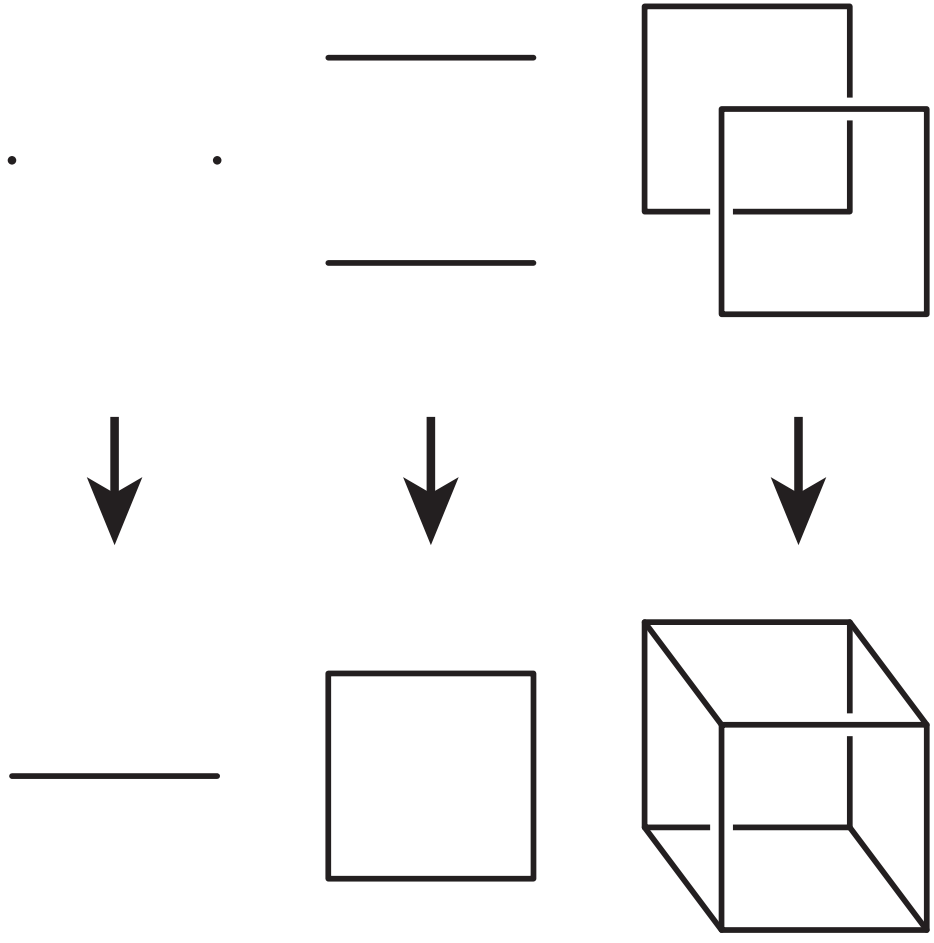

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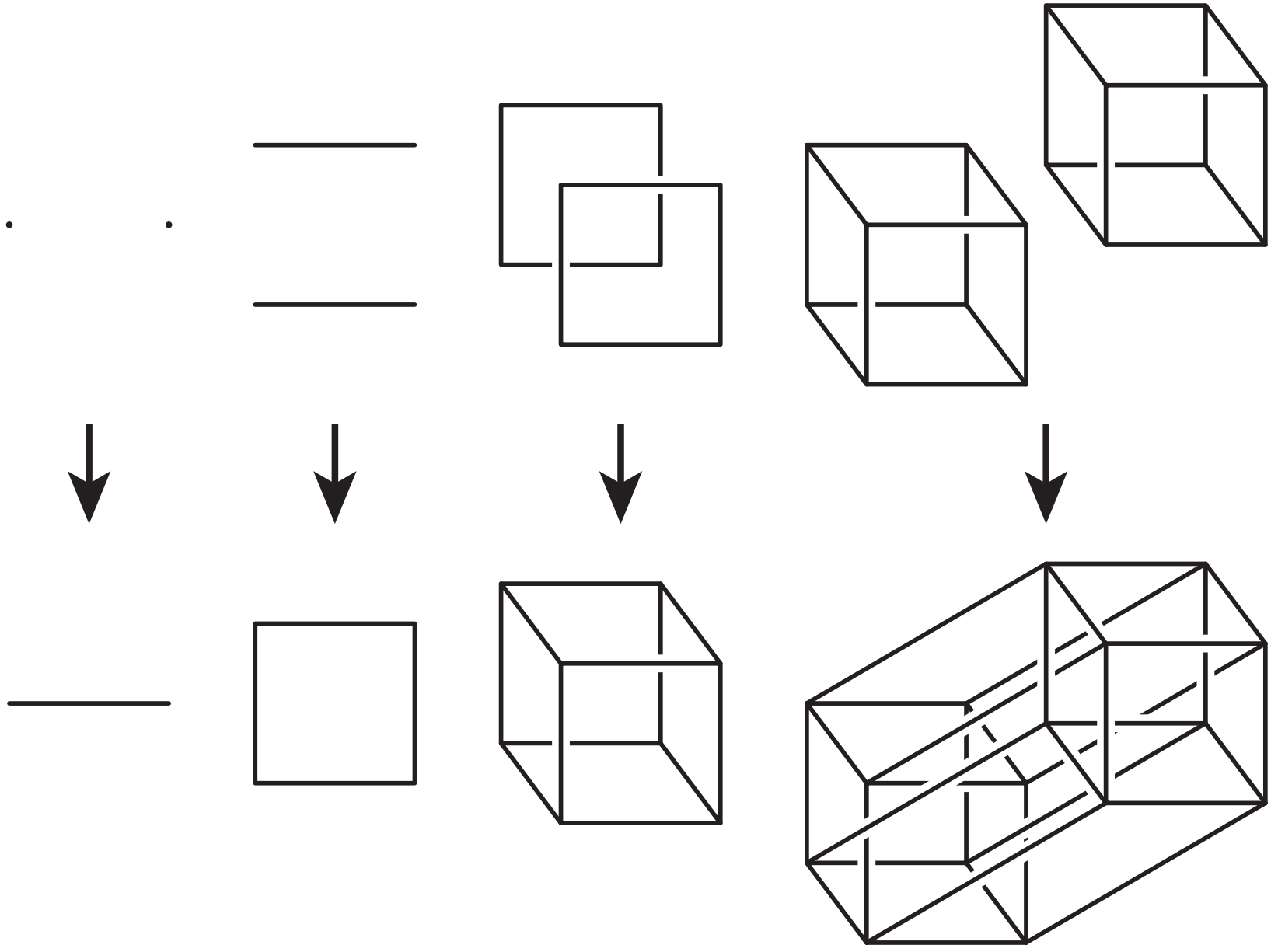

How can we see 4-dimensional things?

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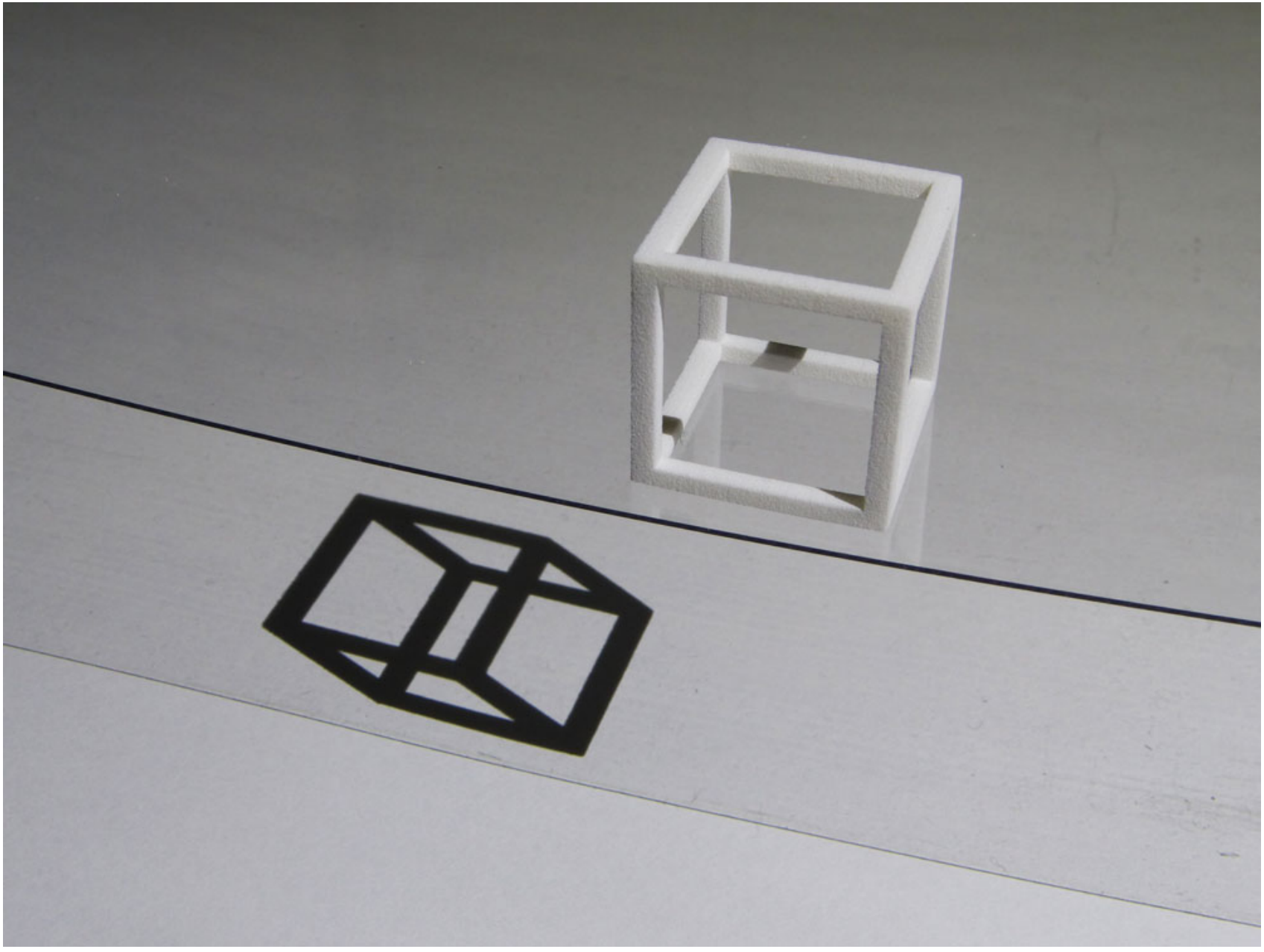
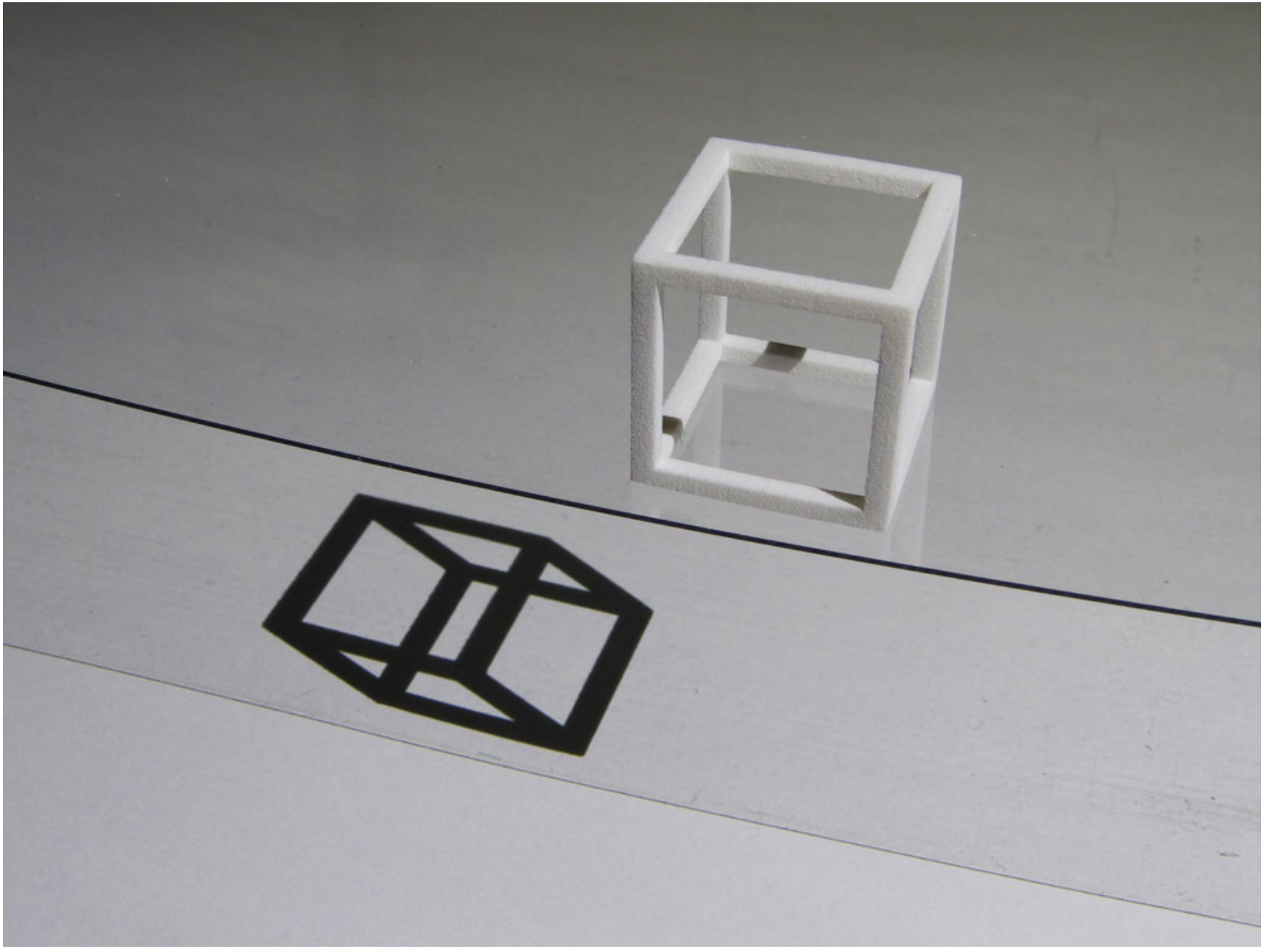
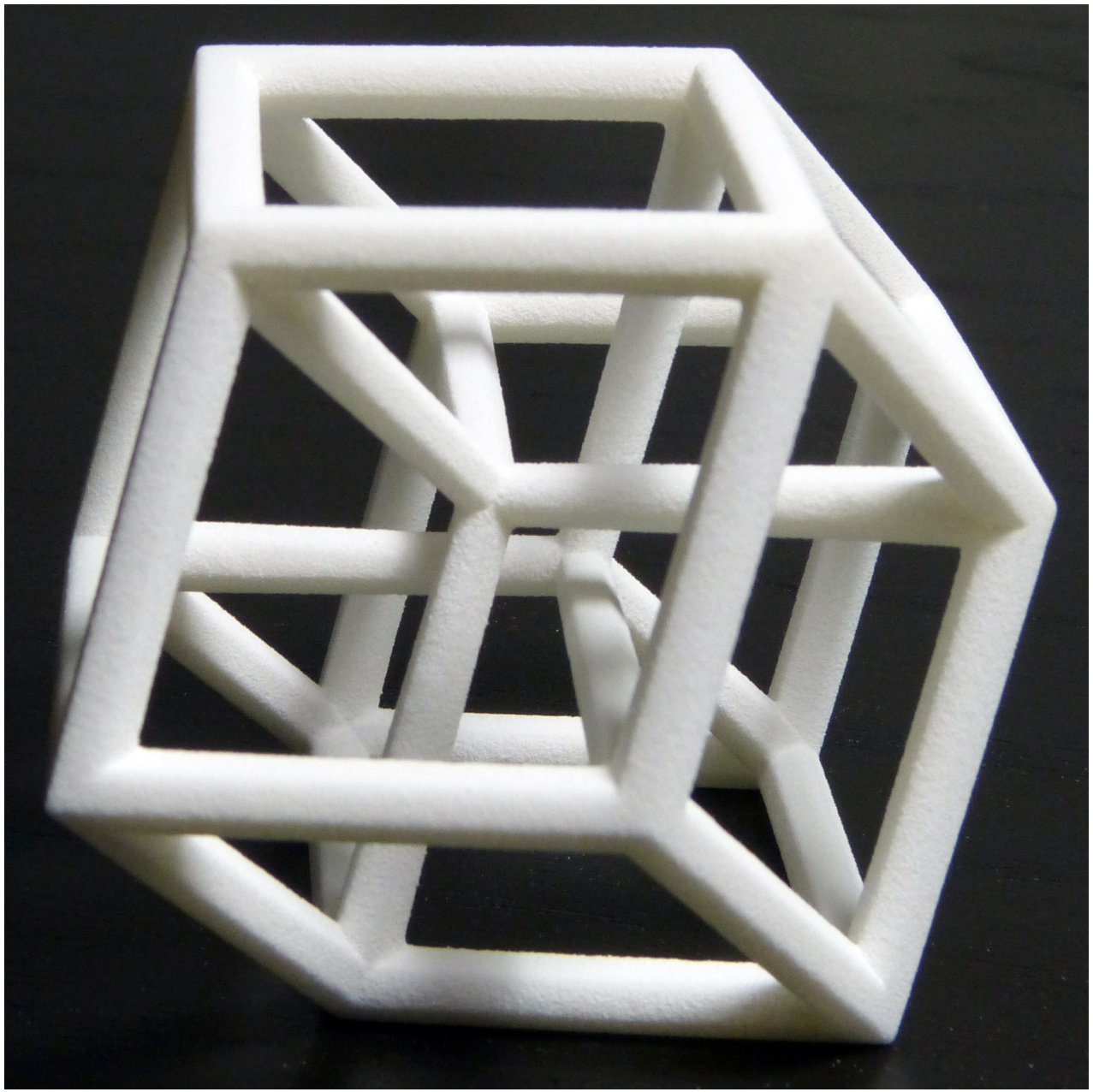


Orthogonal projection of a cube

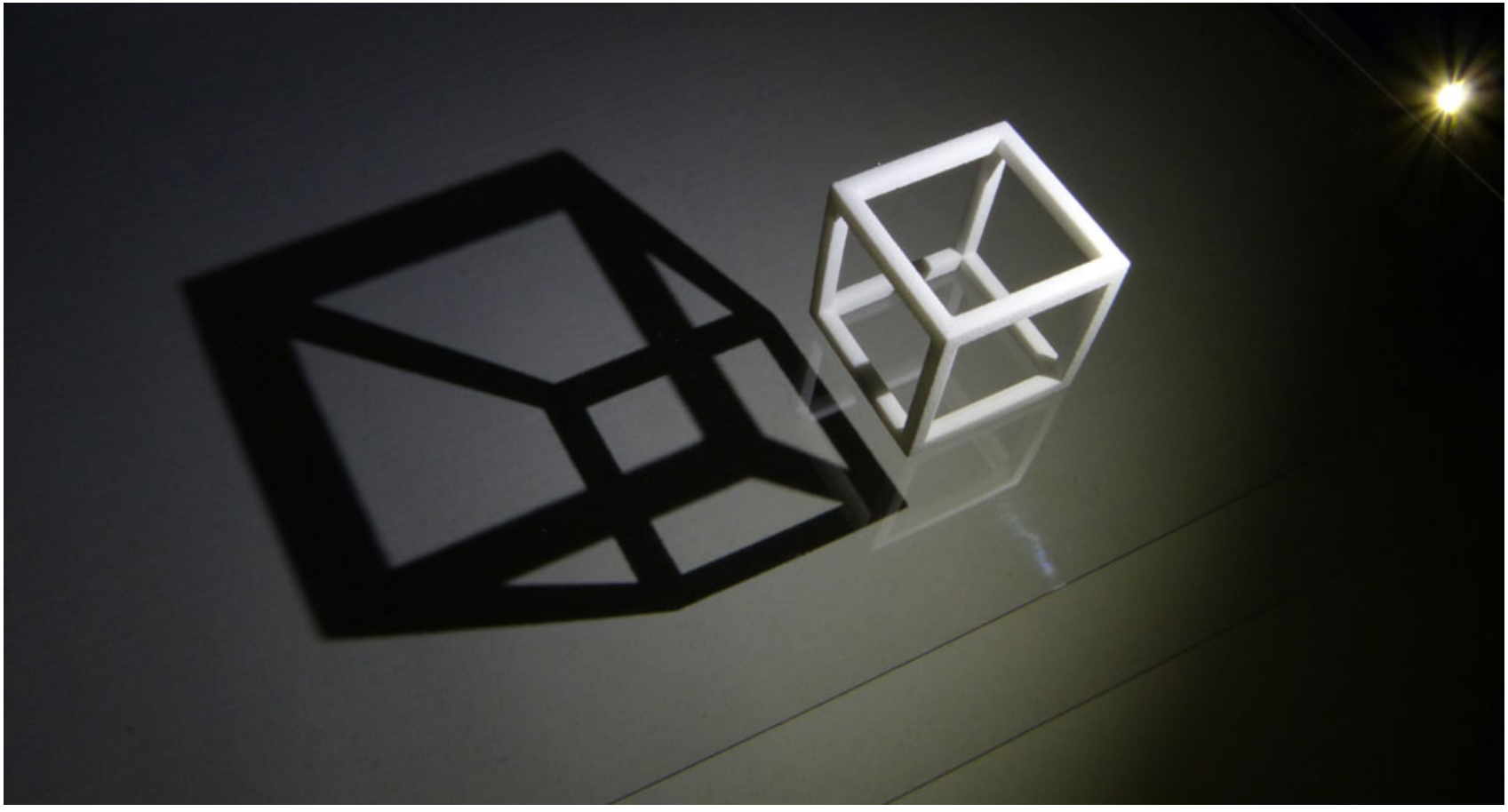


Orthogonal projection of a hypercube

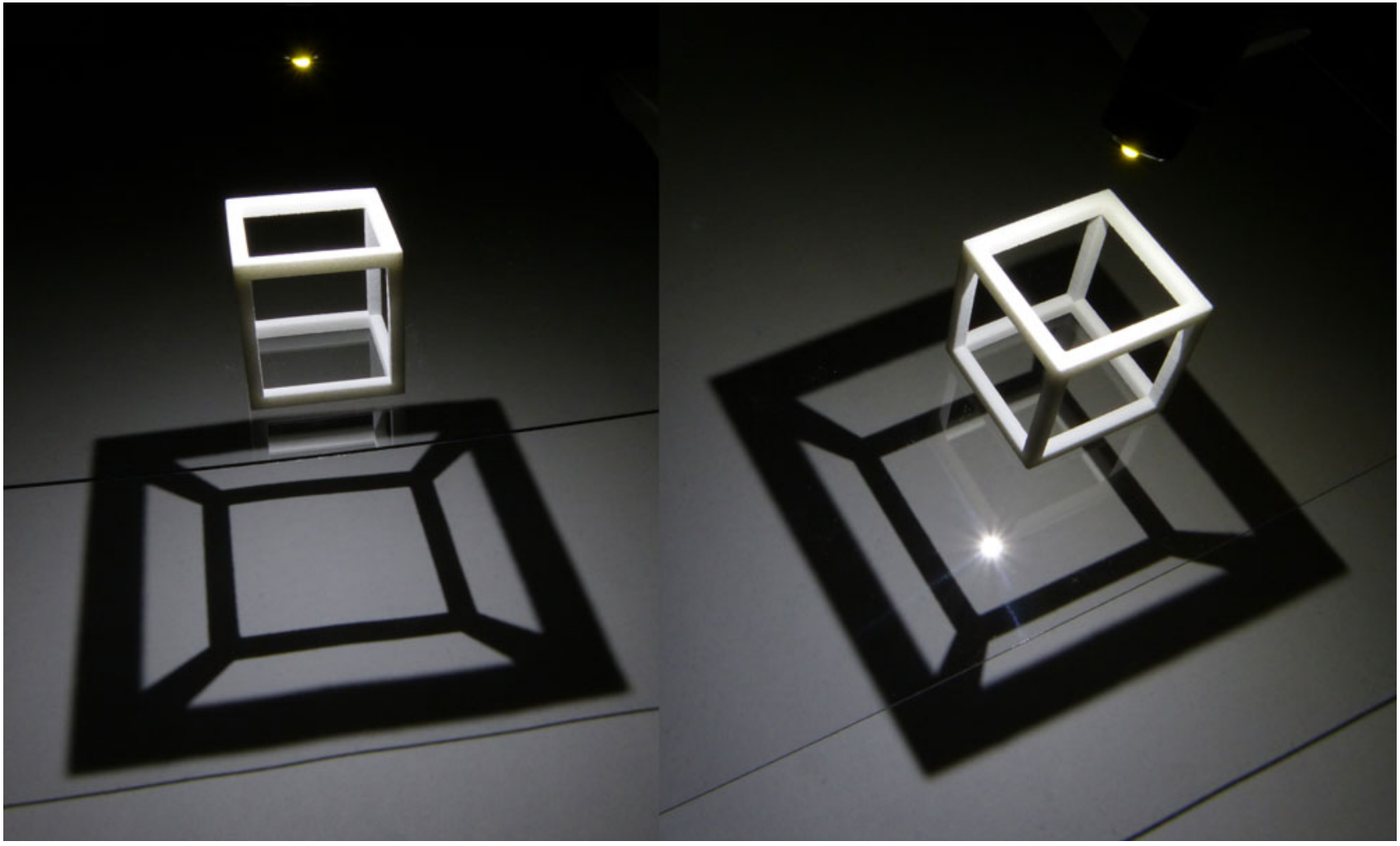


Hypercube B by Bathsheba Grossman.

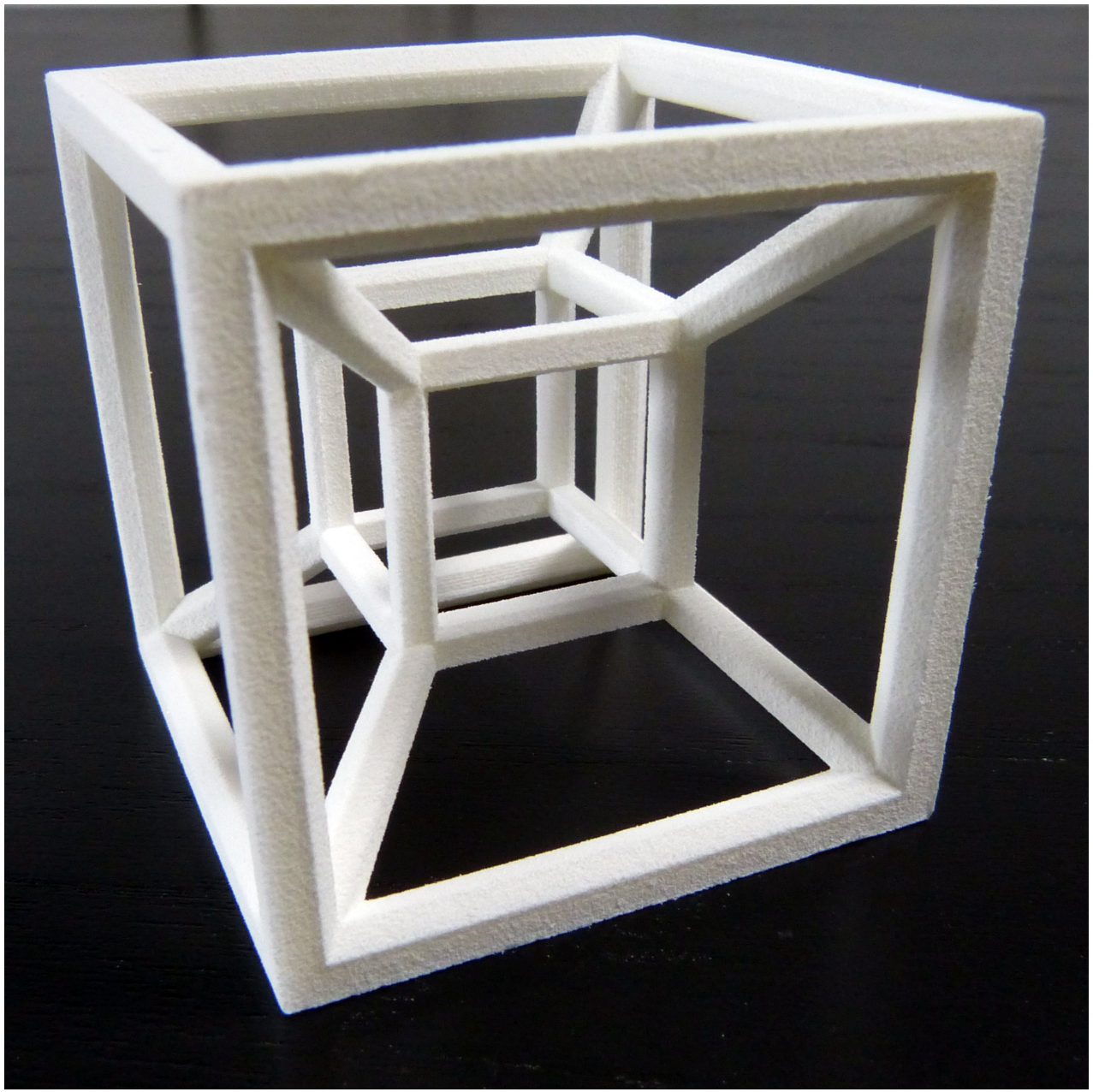
Perspective projection of a cube



Perspective projection of a cube



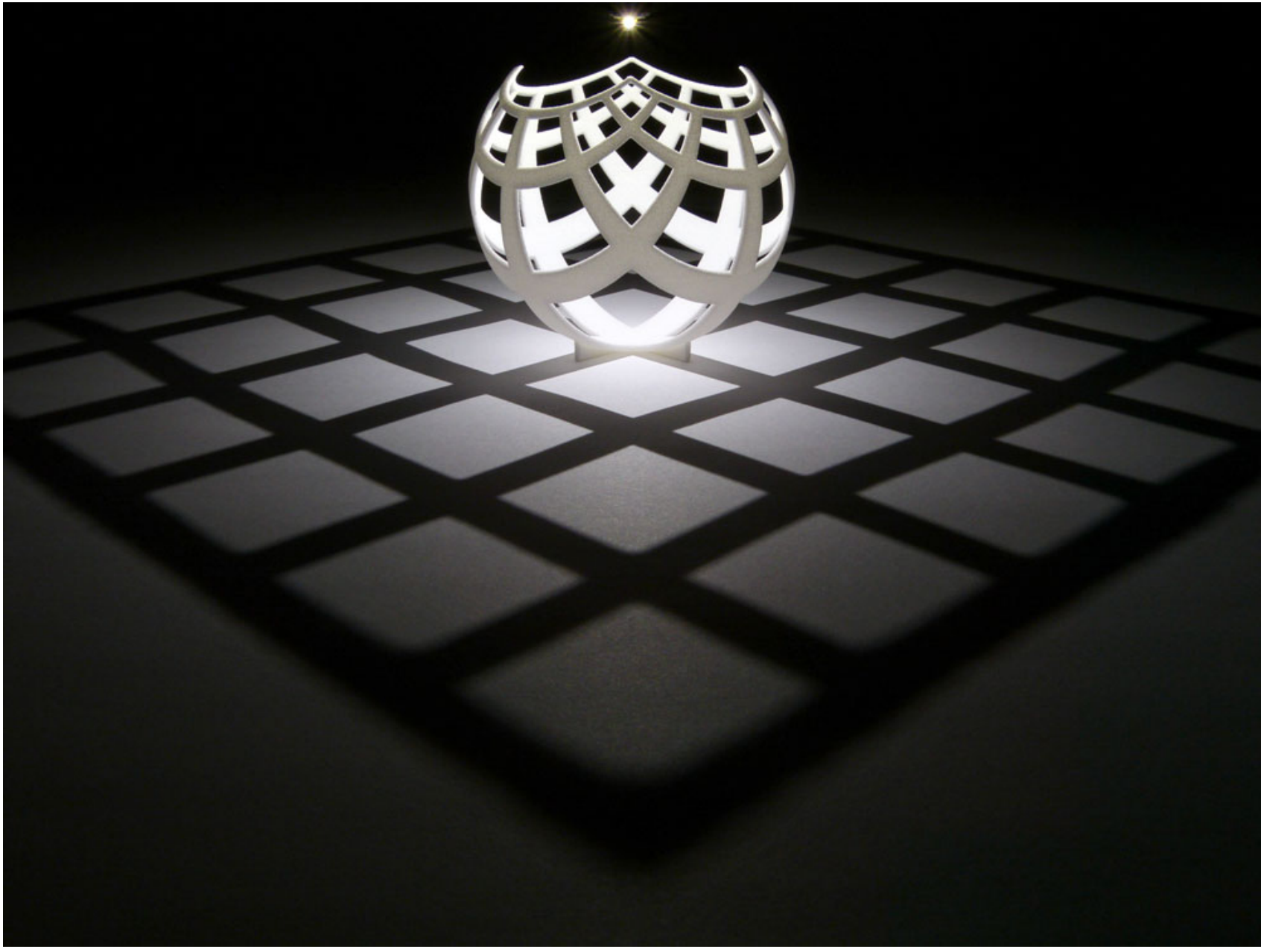
Perspective projection of a hypercube



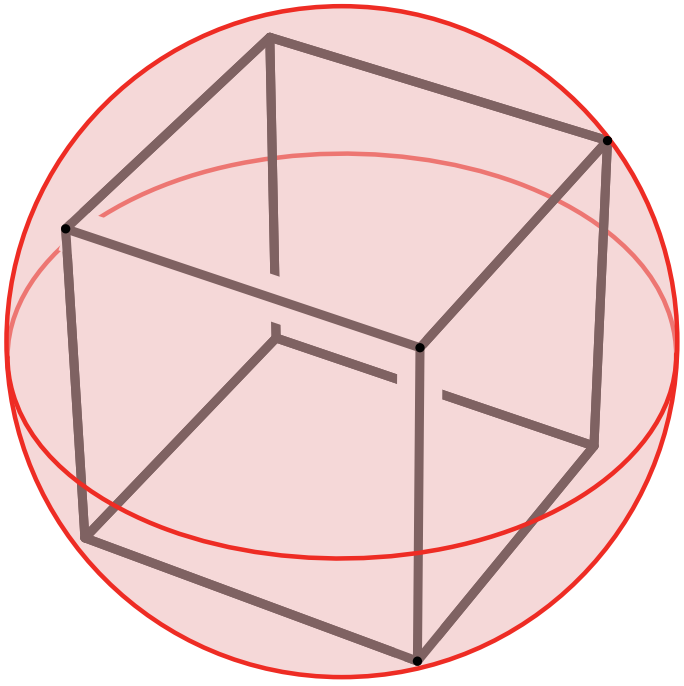
Hypercube A by Bathsheba Grossman.

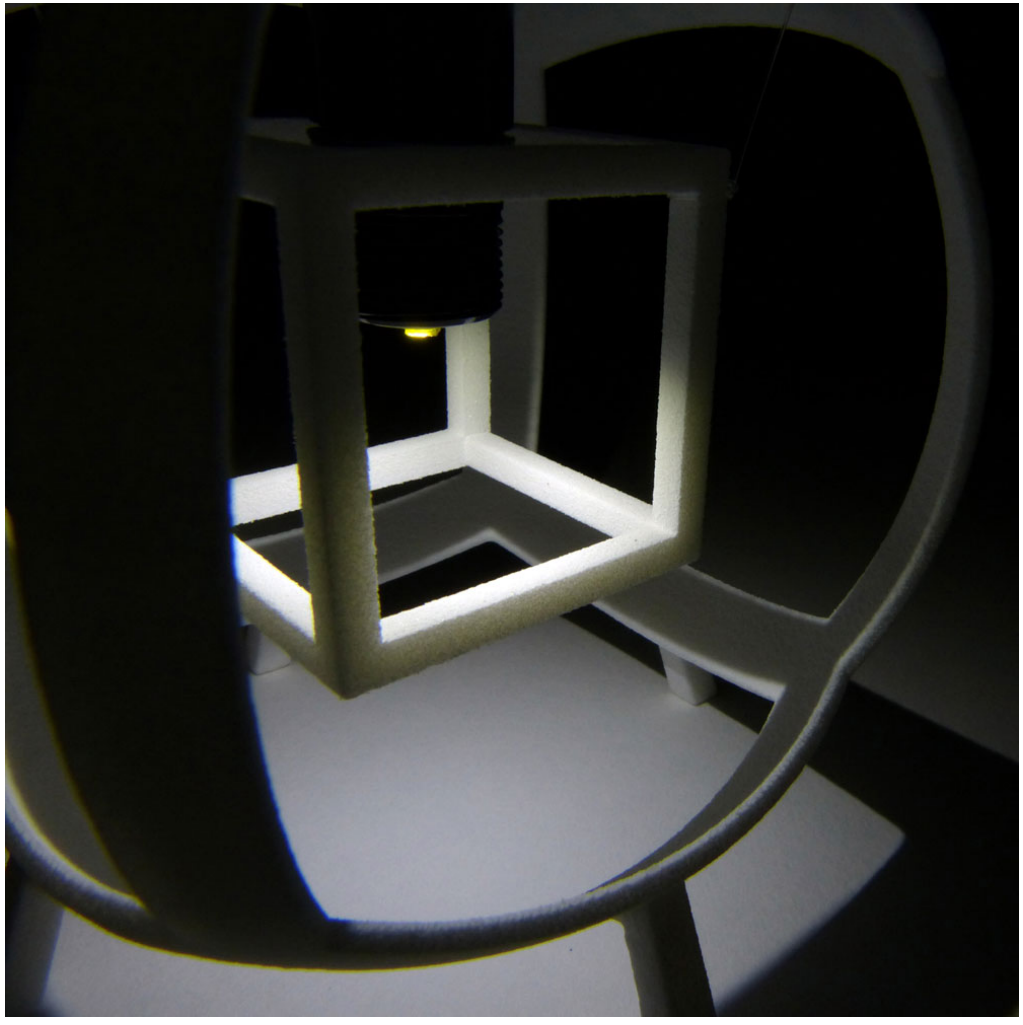
Stereographic projection

Stereographic projection



First radially project the cube to the sphere...





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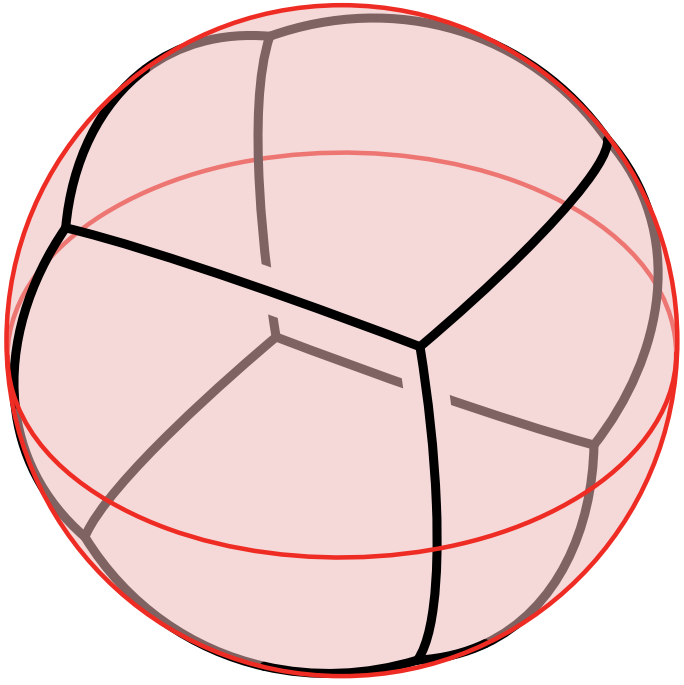
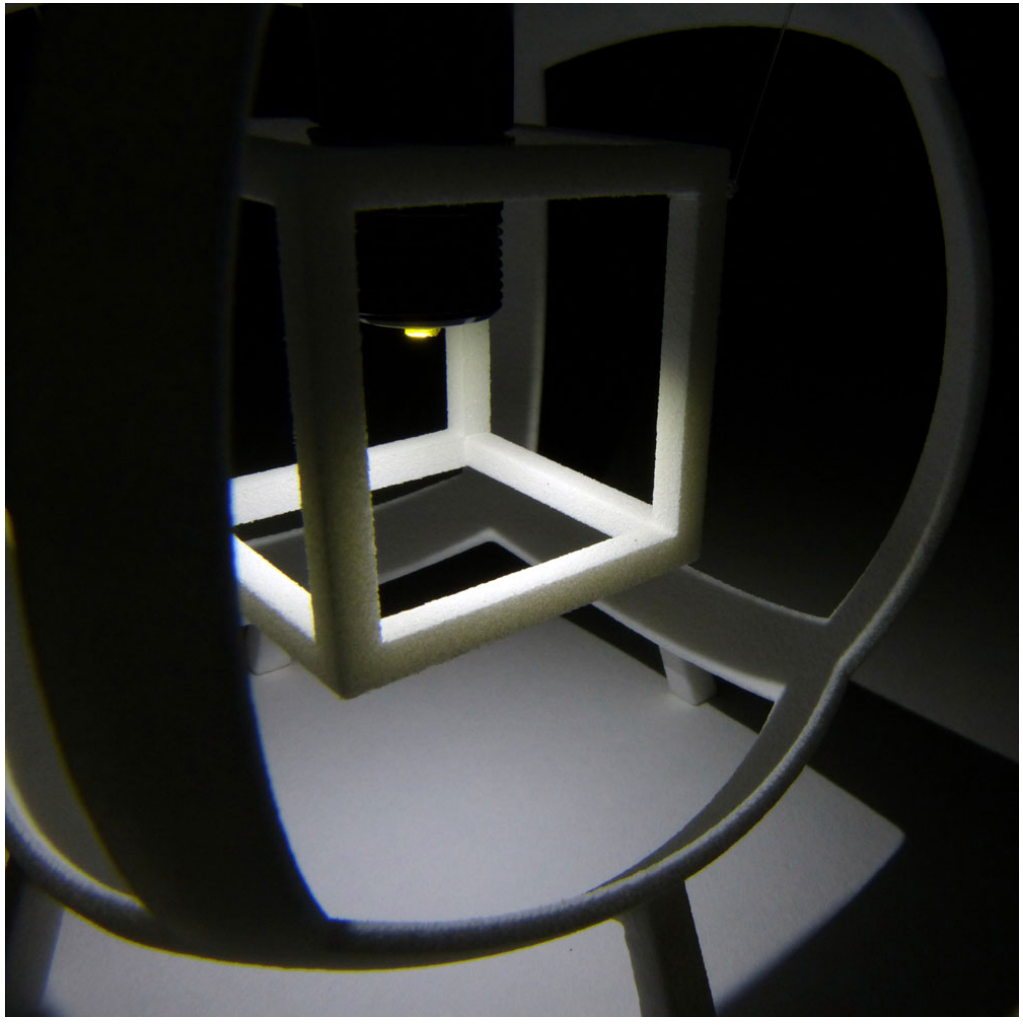
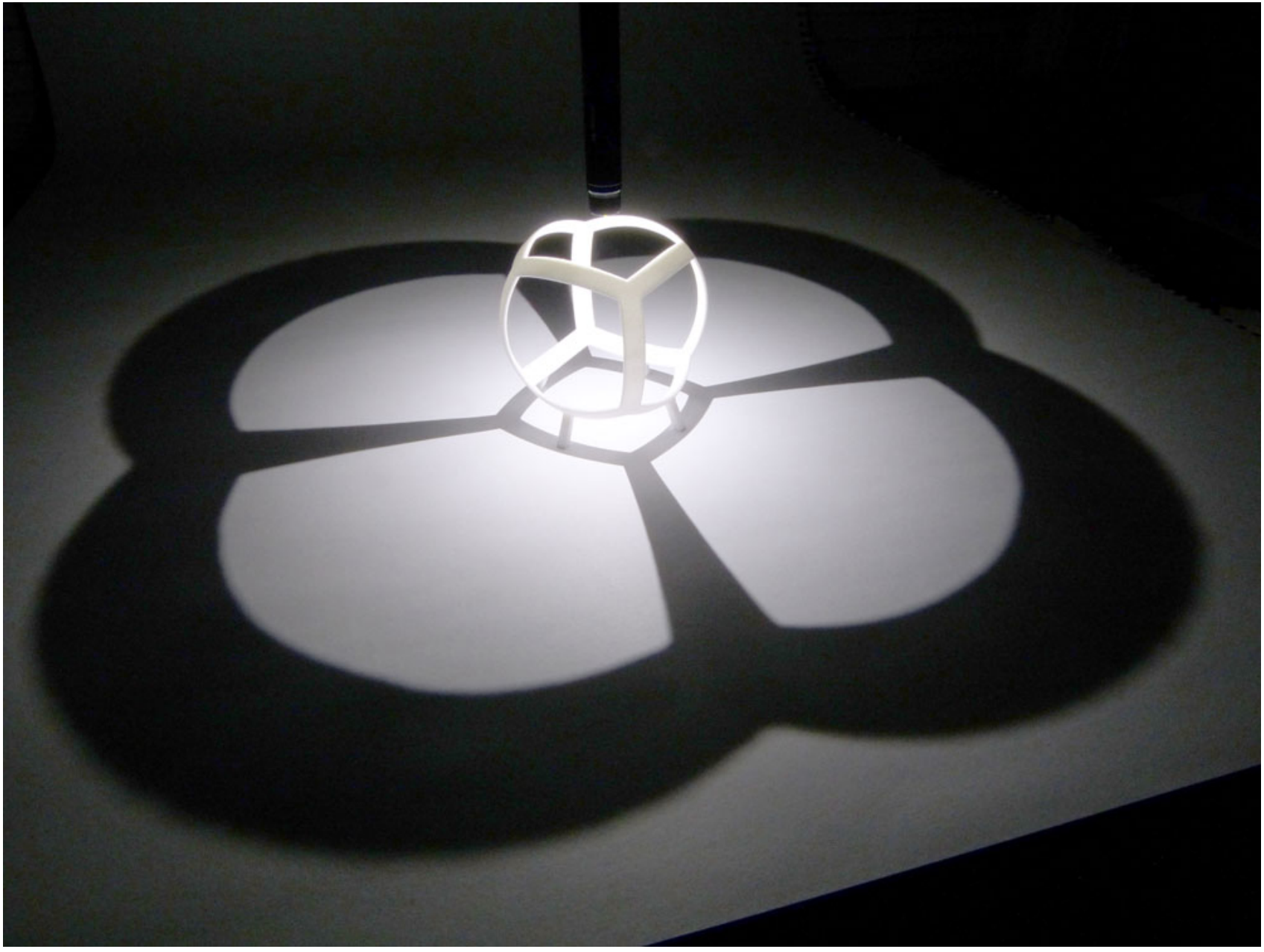




Then stereographically project to the plane



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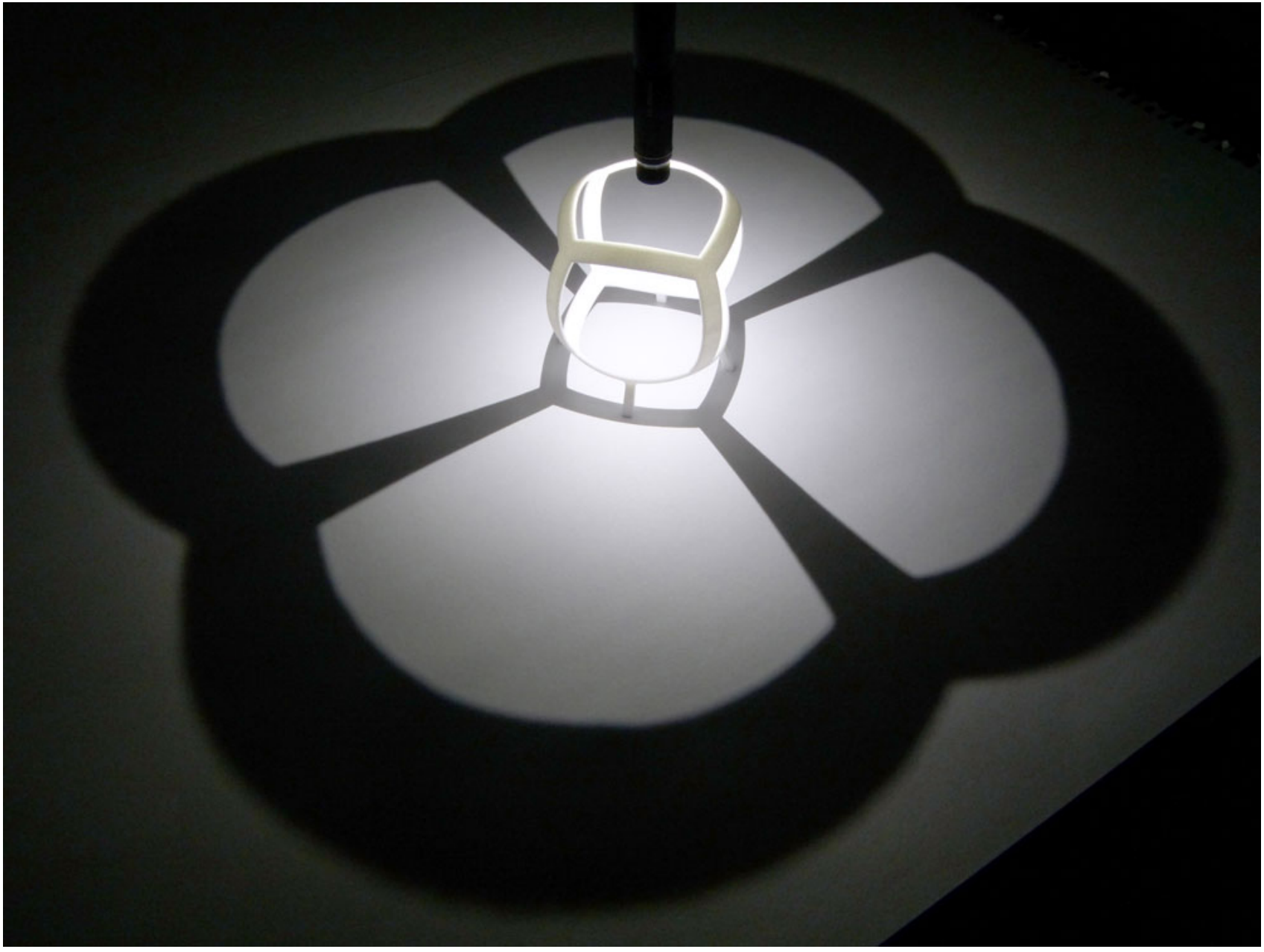
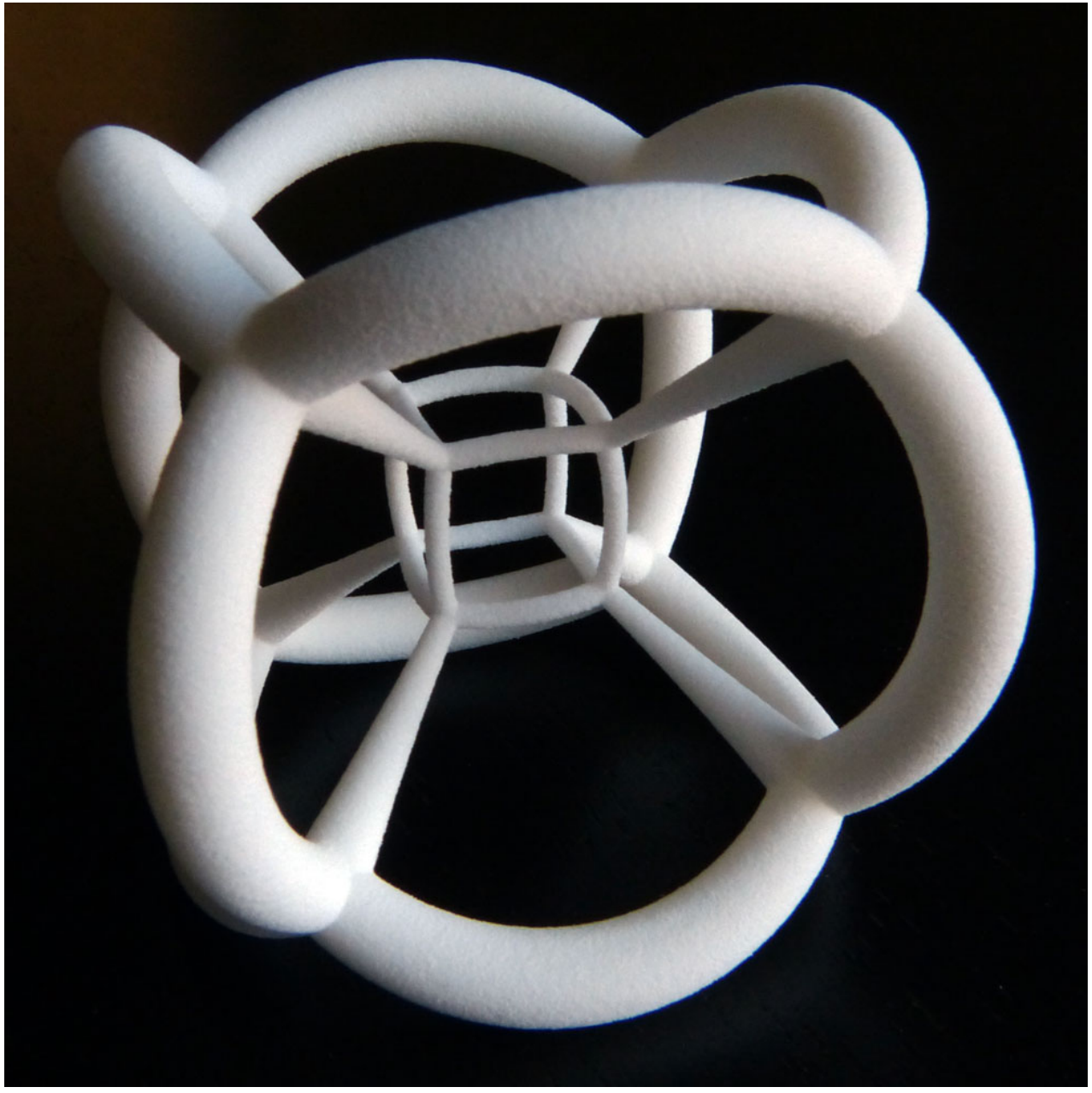


Do the same thing one dimension up to see a hypercube



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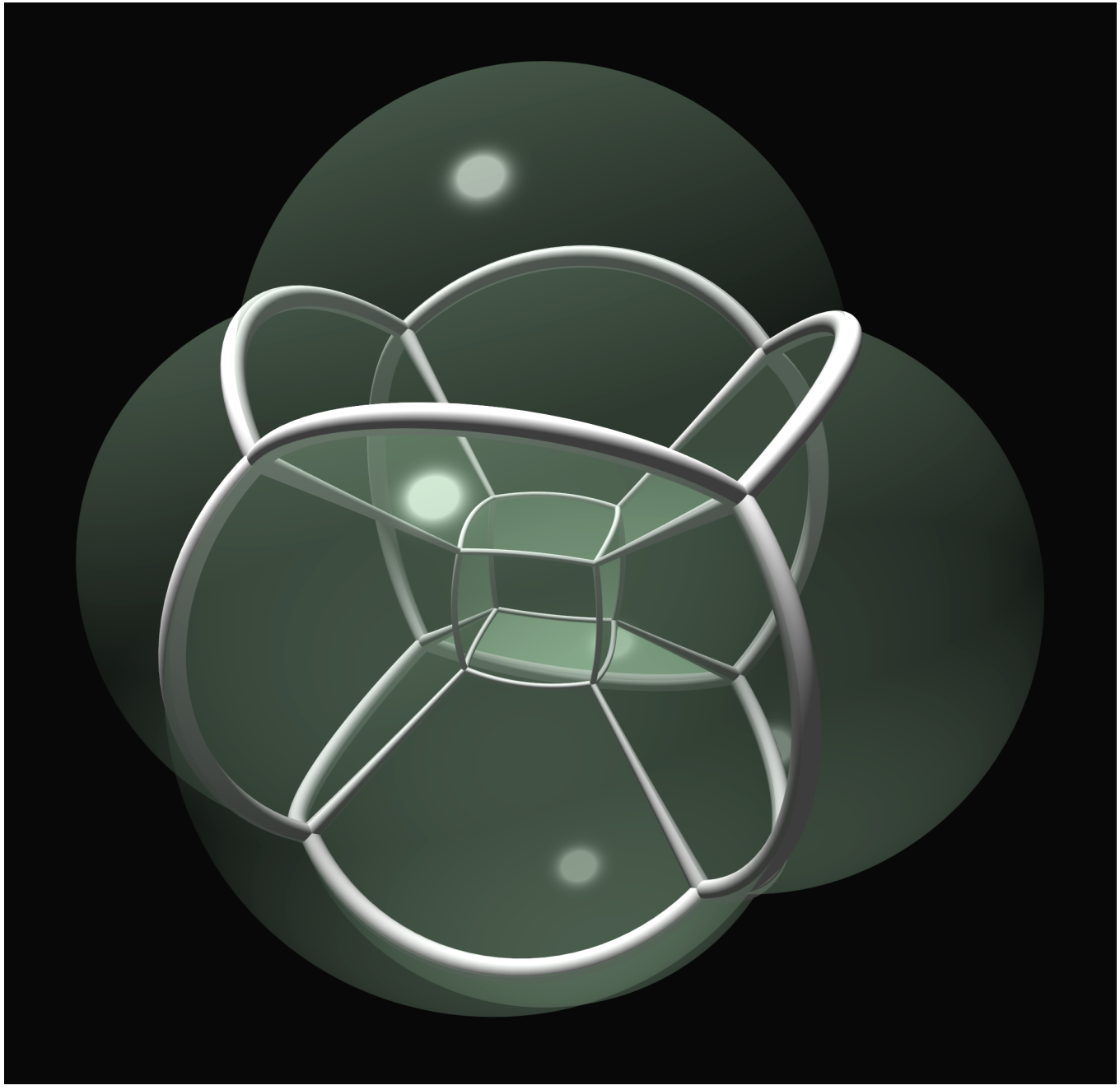
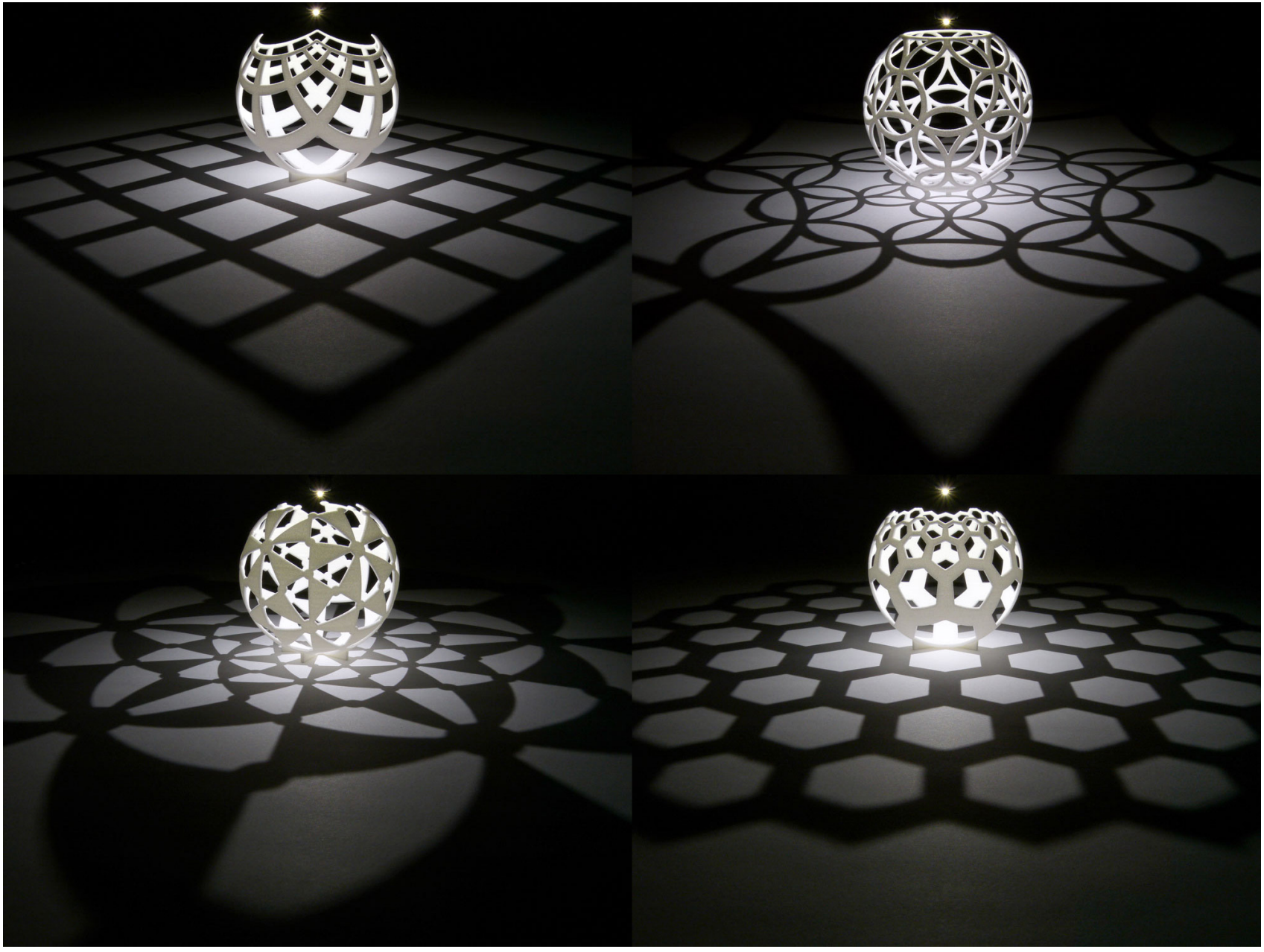


More amazing properties of stereographic projection



Visualising the sphere in 4-dimensional space

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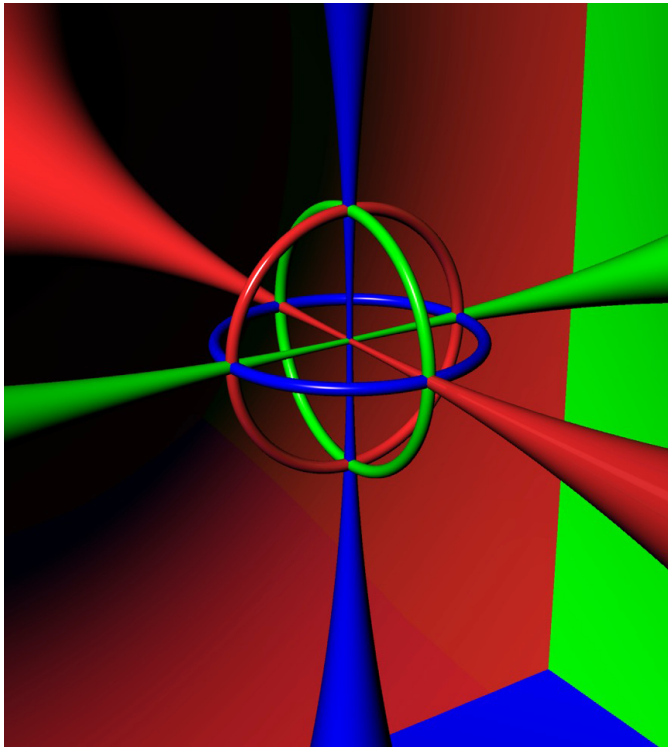
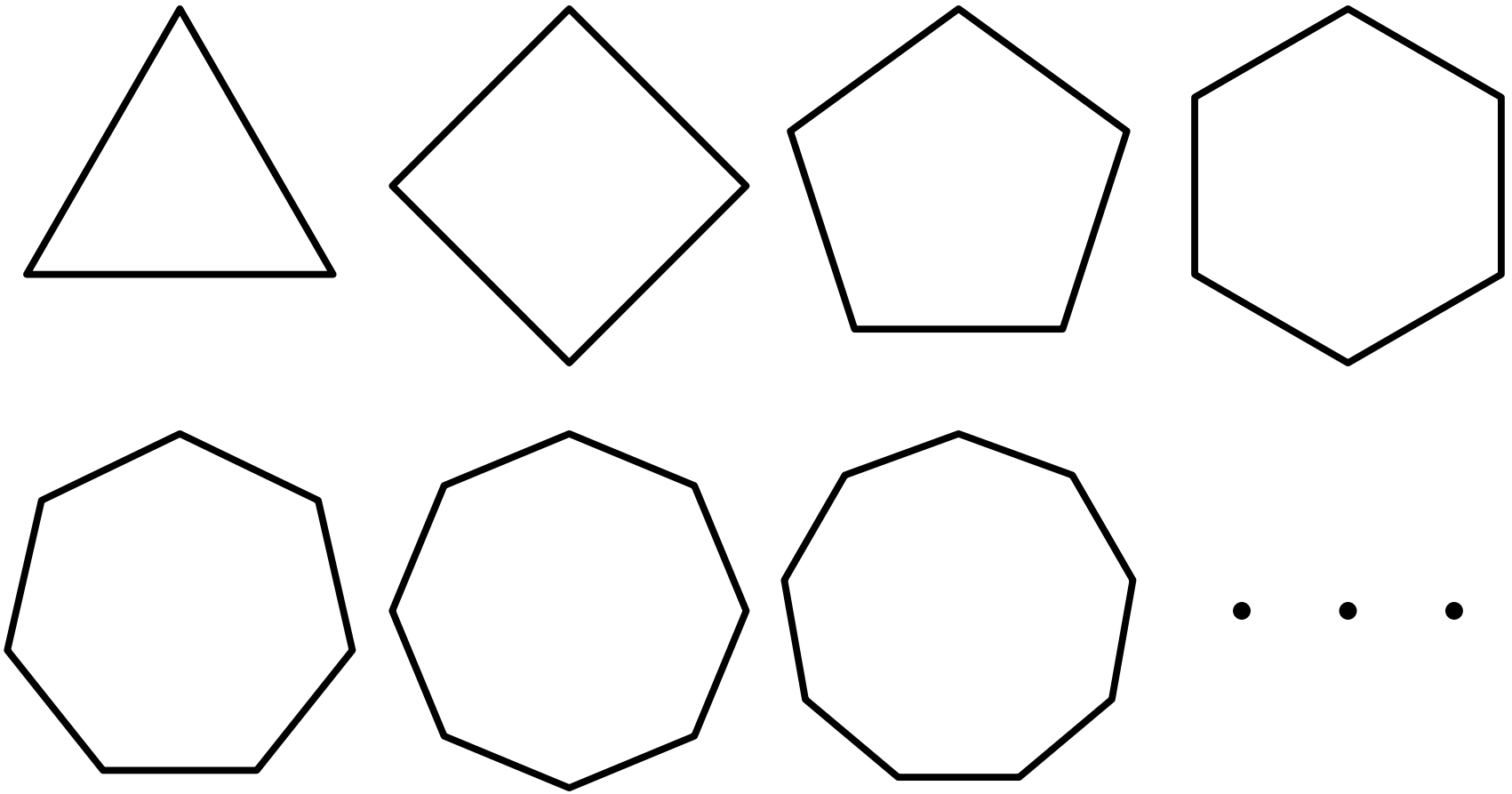
- The sphere in 4-dimensional space is "the same as" 3-dimensional space, plus a point.



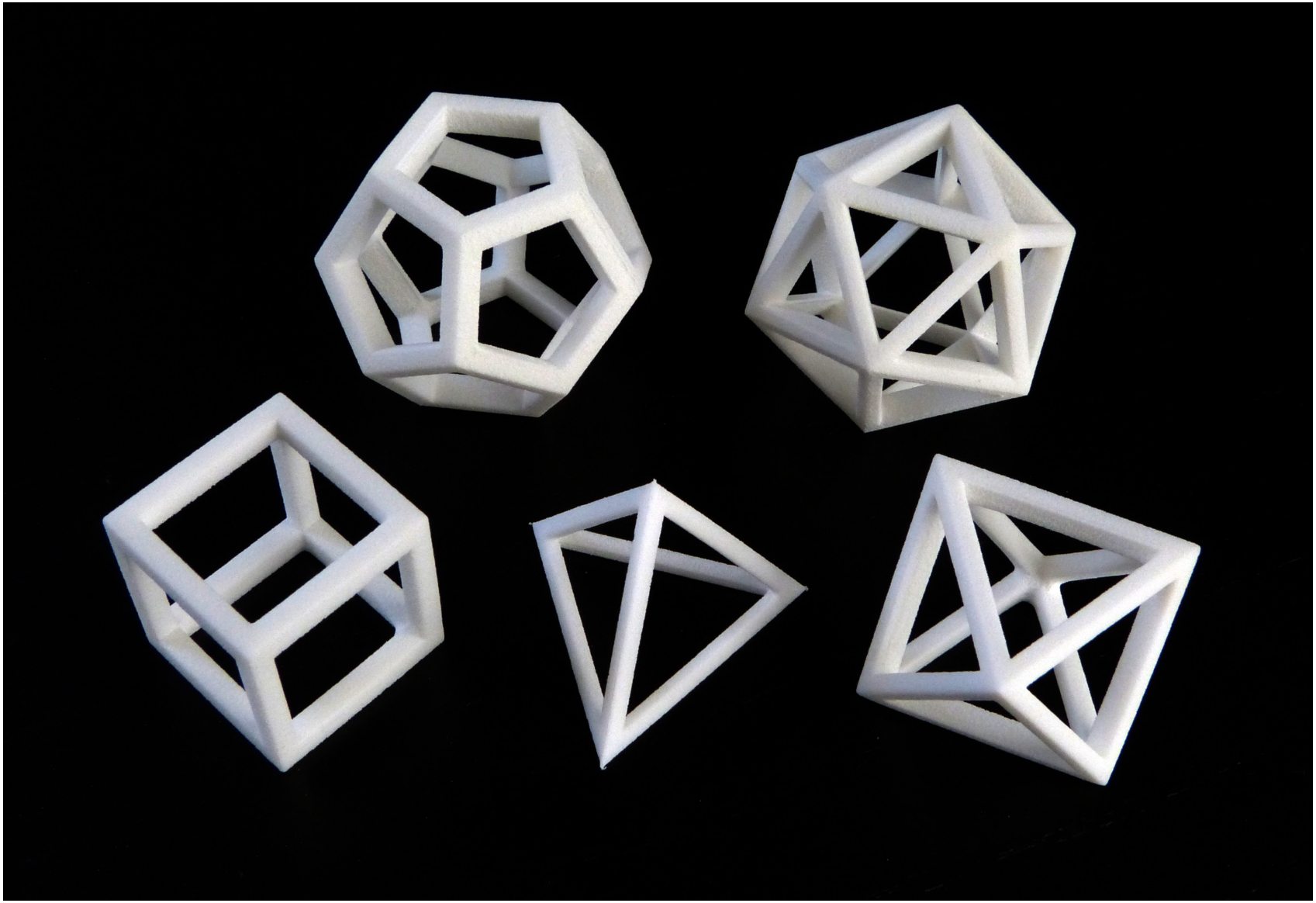
Regular Polytopes

In 2-dimensions: Regular polygons

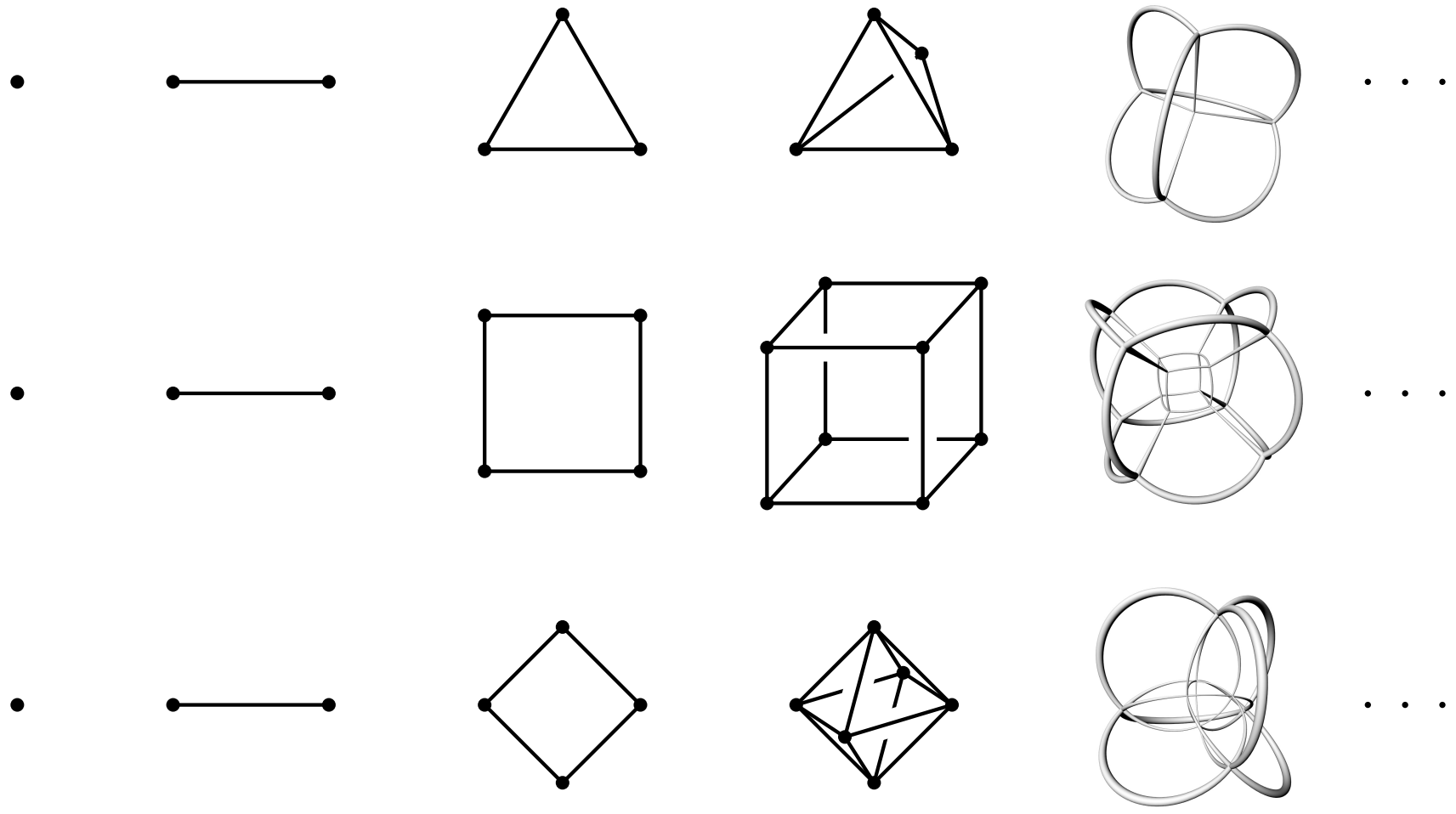


Regular Polytopes

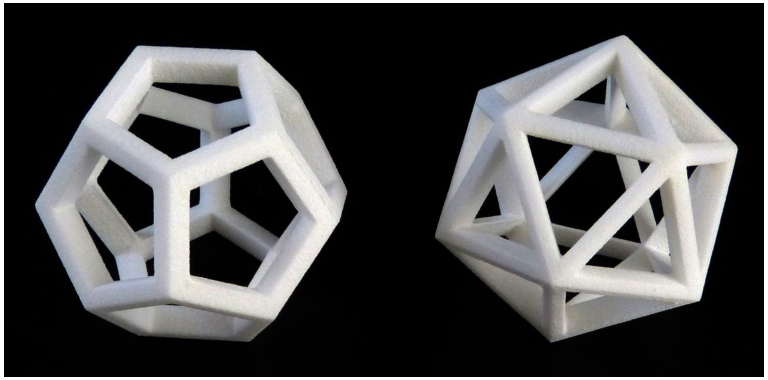
In 3-dimensions: Regular polyhedra



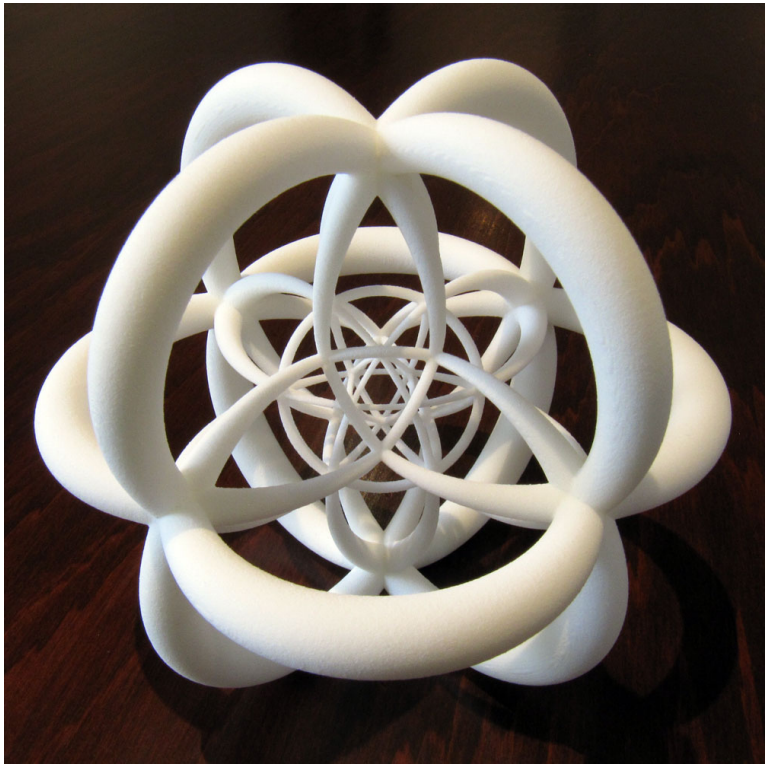
Three families of regular polytopes



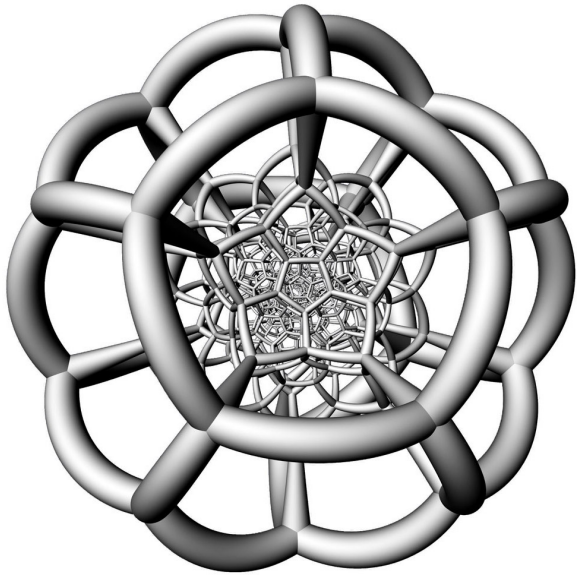
The only exceptions!



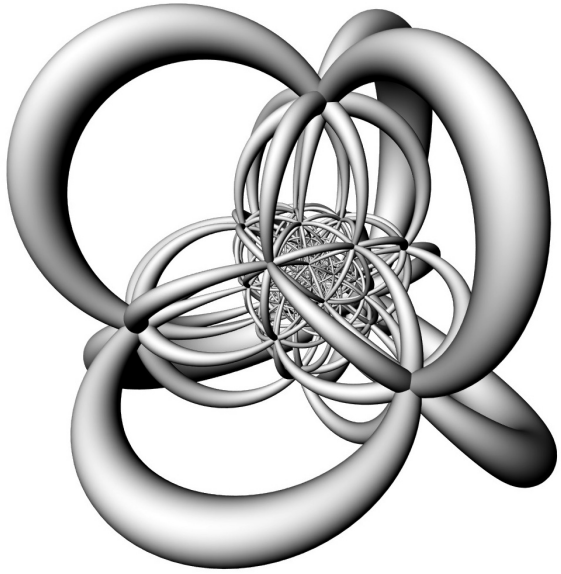
Dodecahedron Icosahedron



24-cell

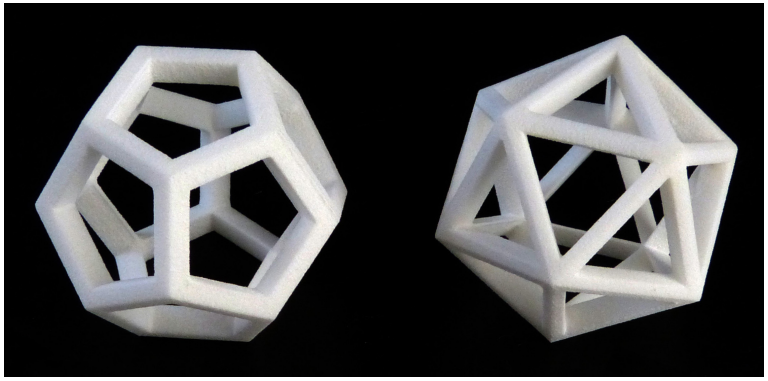


120-cell



600-cell

The only exceptions!



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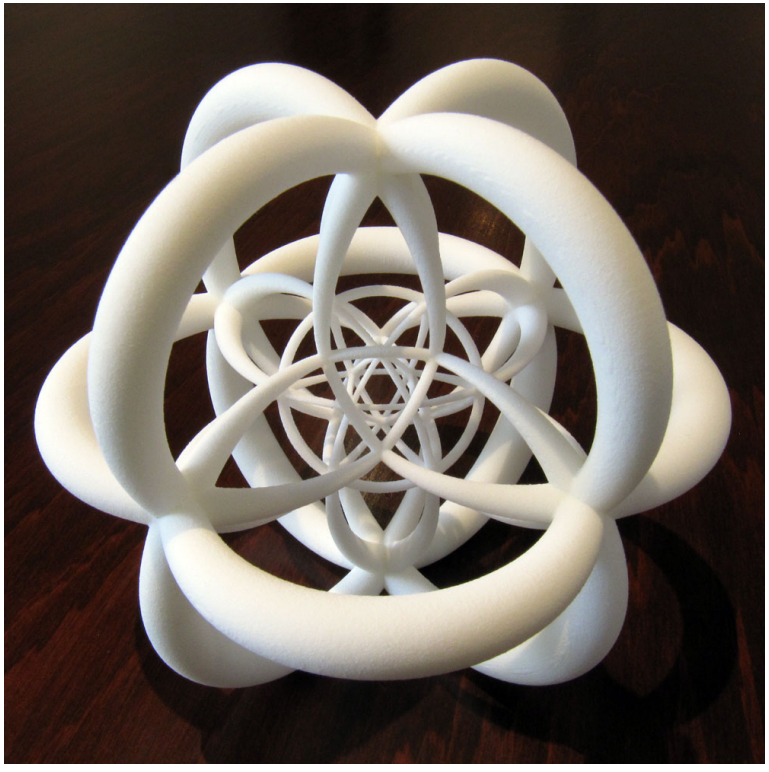
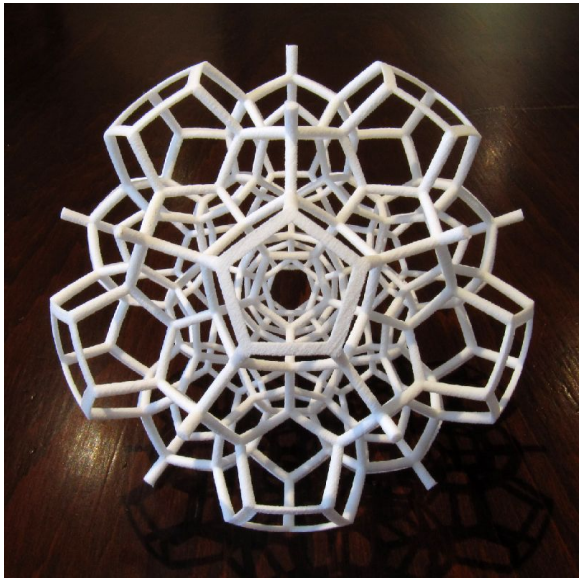
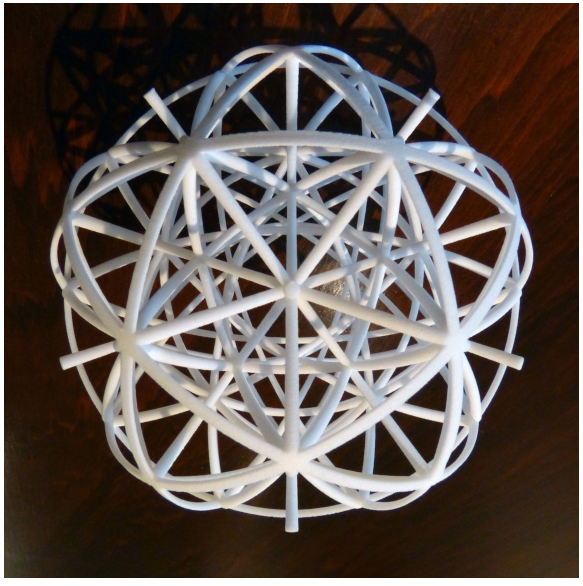


24-cell

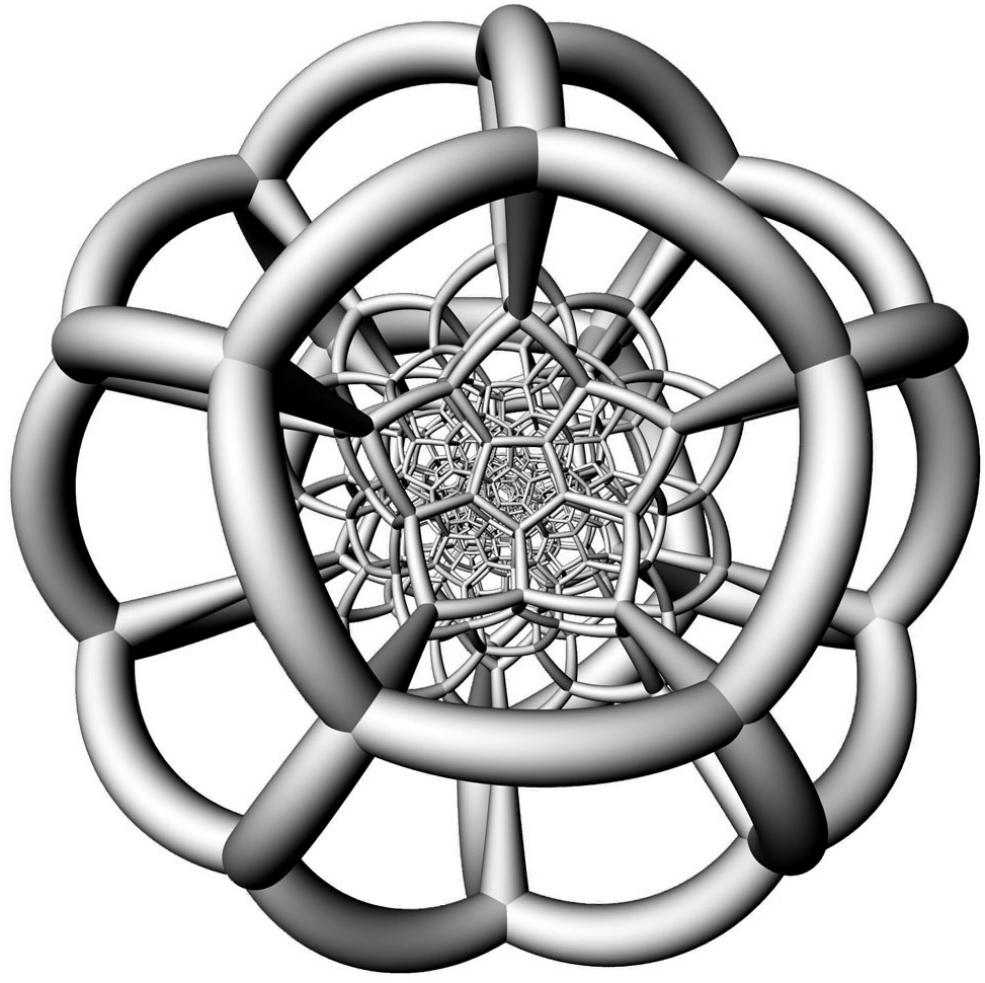


Half of a 120-cell



Half of a 600-cell

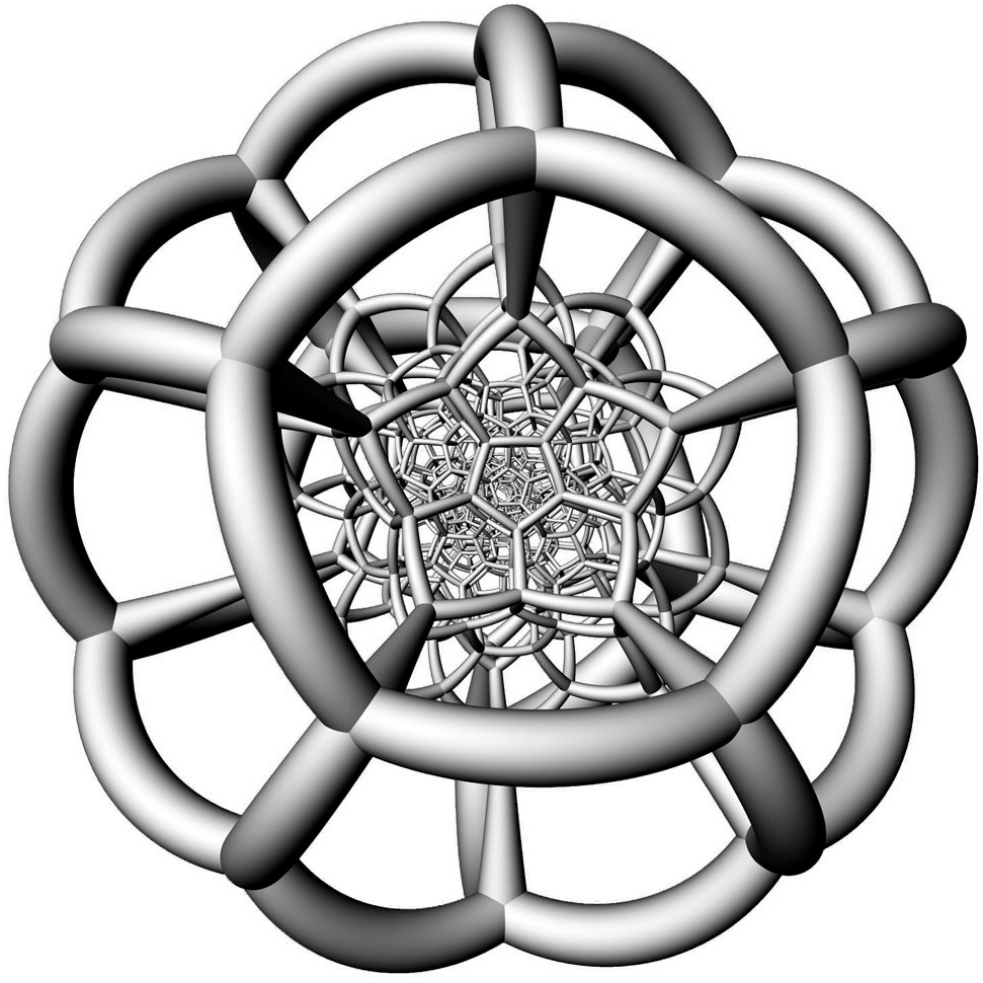
Puzzling the 120-cell



Puzzling the 120-cell

The 120–cell has

- 120 dodecahedral cells,
- 720 pentagonal faces,
- 1200 edges, and
- 600 vertices.



Spherical layers in the 120–cell

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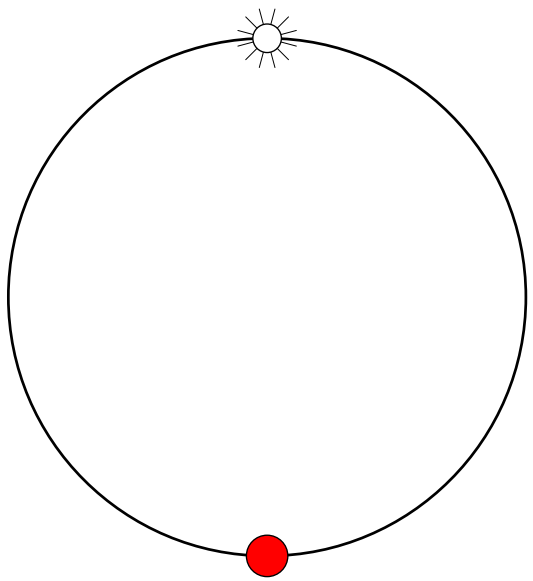

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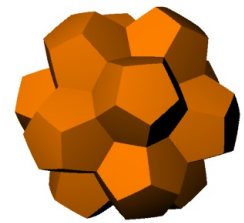
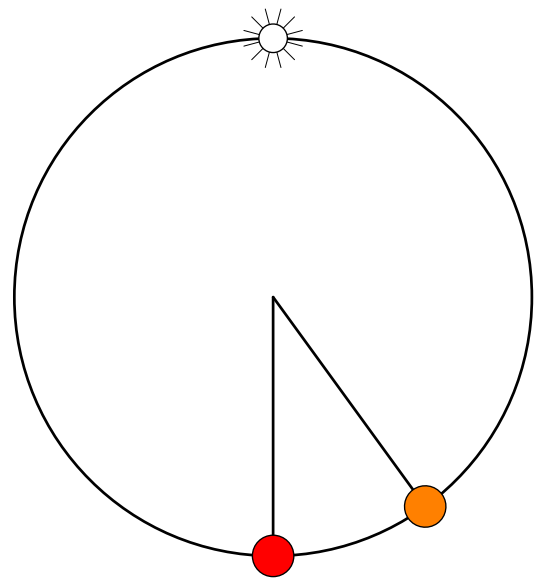
One way to understand the 120–cell is to look at the layers of dodecahedra around the central dodecahedron.

- 1 central dodecahedron
- 12 dodecahedra at angle $\pi/5$





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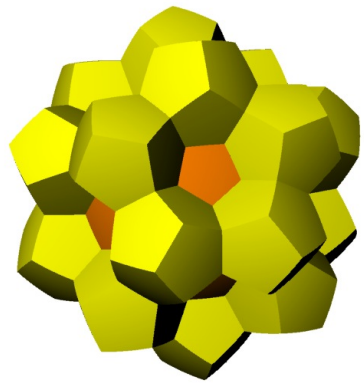
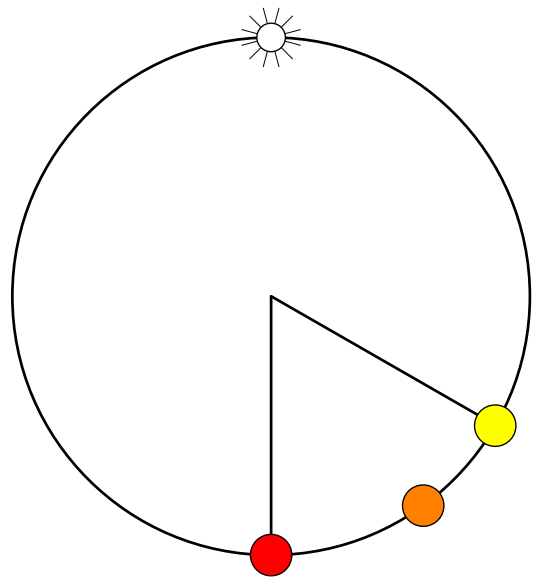
- 20 dodecahedra at angle $\pi/3$





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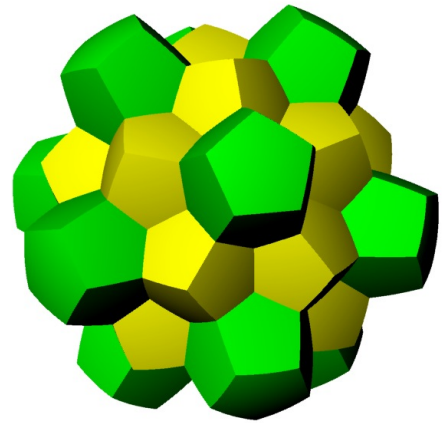

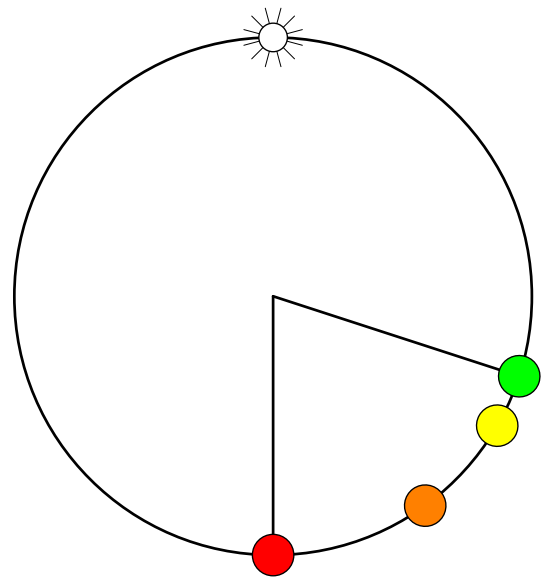

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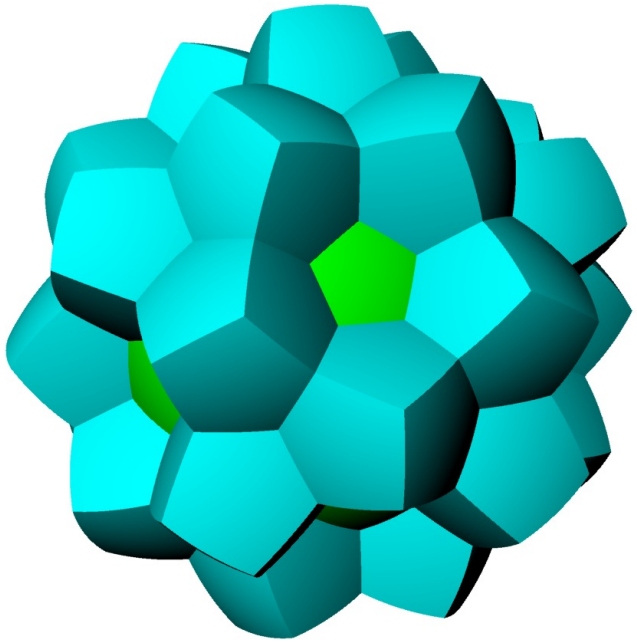
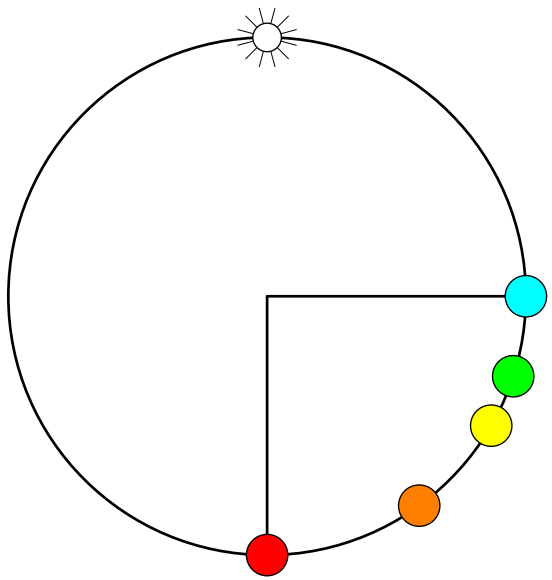
- 12 dodecahedra at angle $2\pi/5$
- 30 dodecahedra at angle $\pi/2$





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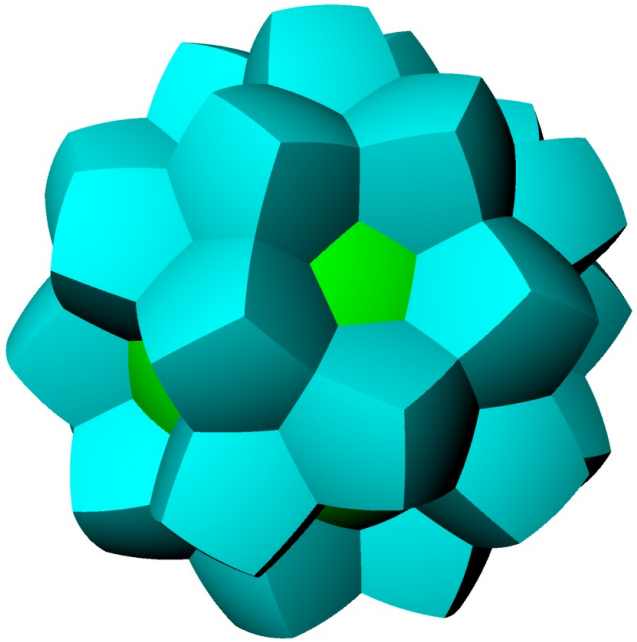
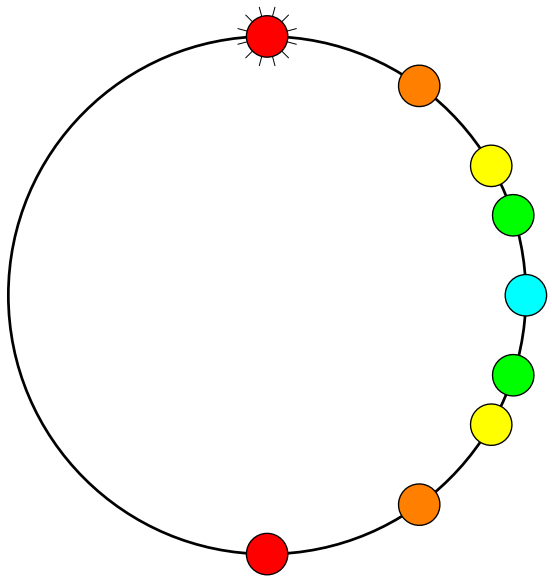
The pattern is mirrored in the last four layers.

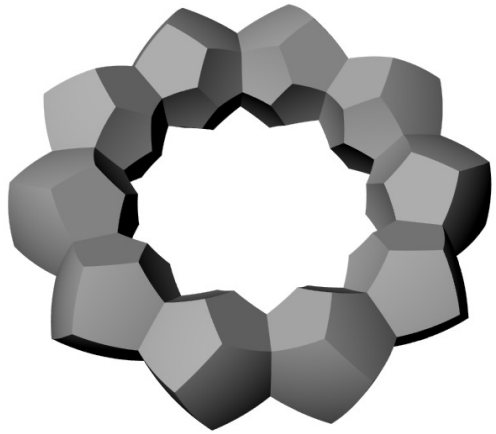
$1+12+20+12+30+12+20+12+1 = 120$





Rings of dodecahedra in the 120–cell

A second way to understand the 120–cell is by making it up out of rings of 10 dodecahedra.



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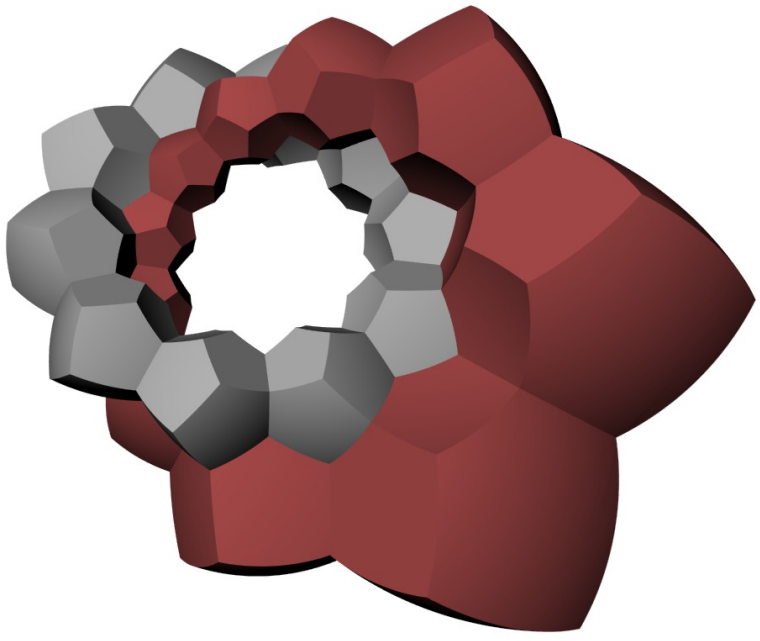

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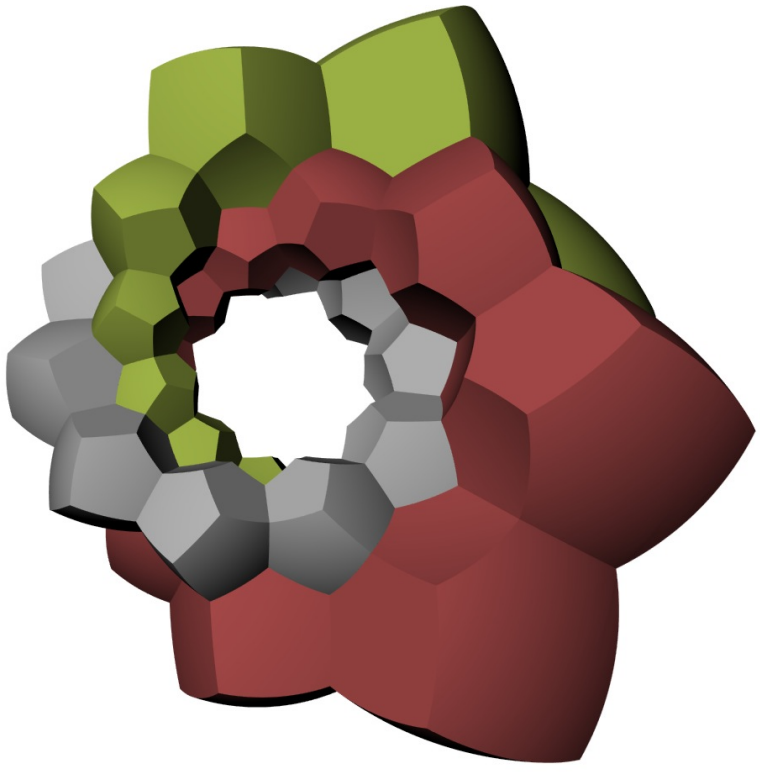

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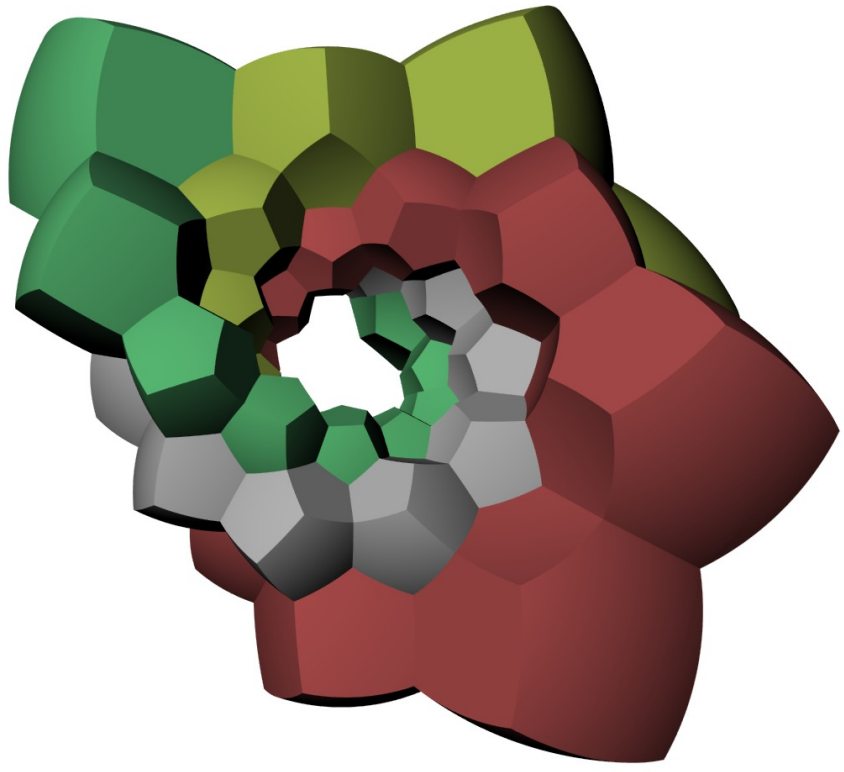

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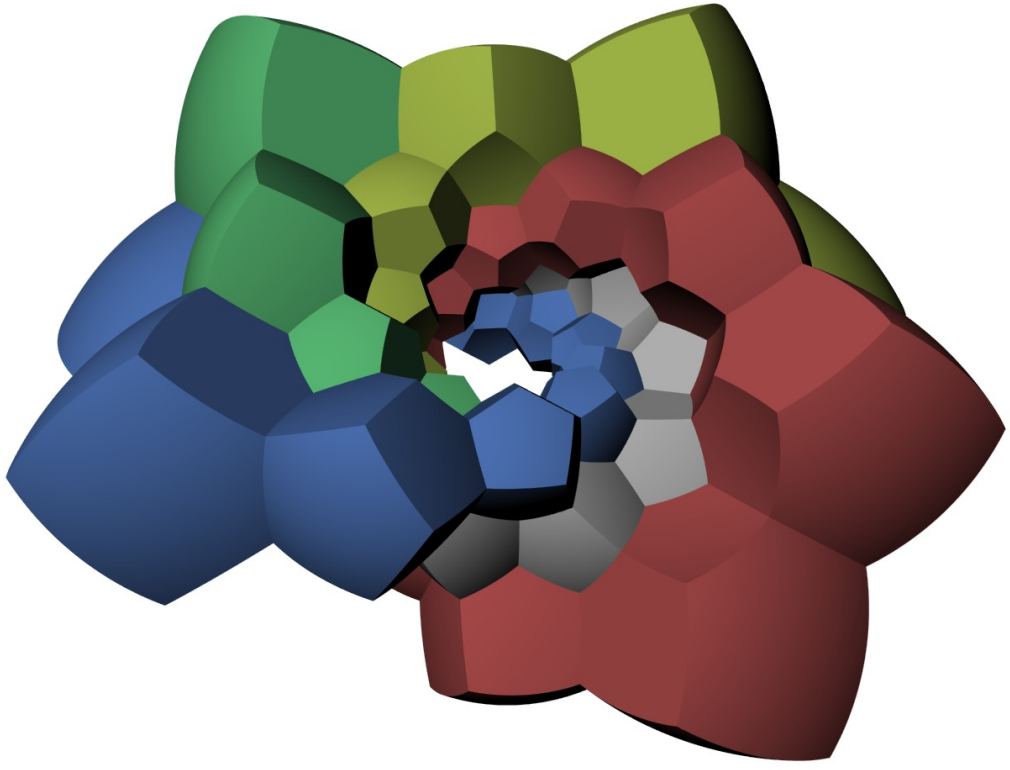

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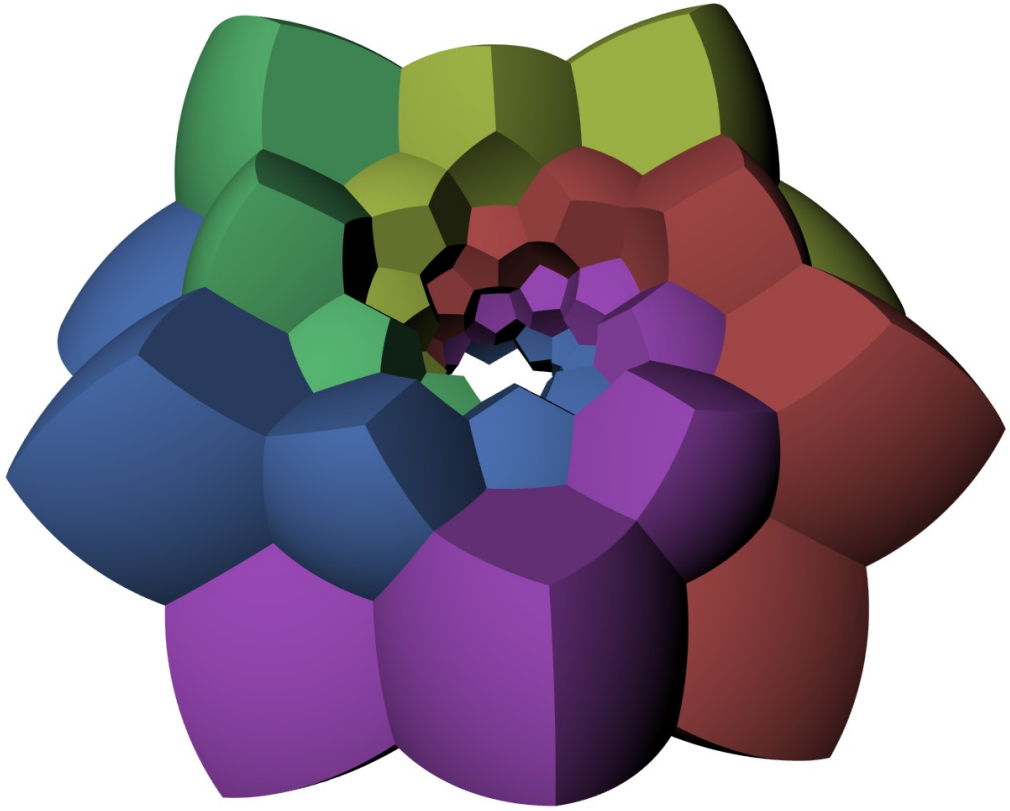

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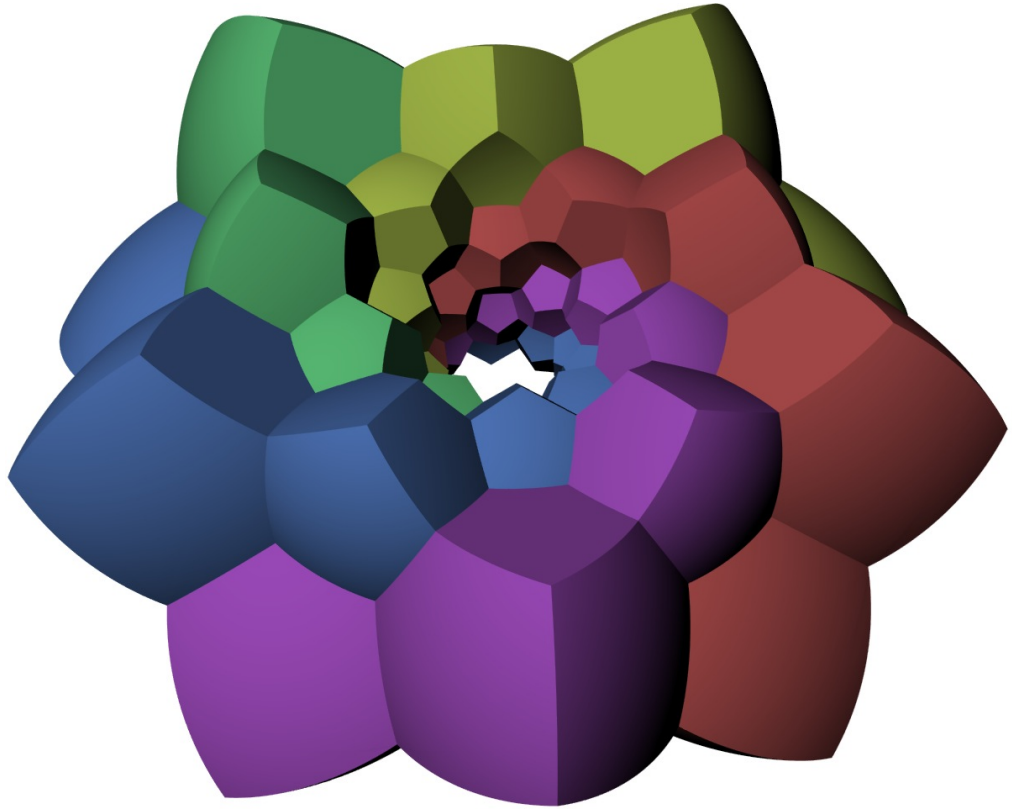

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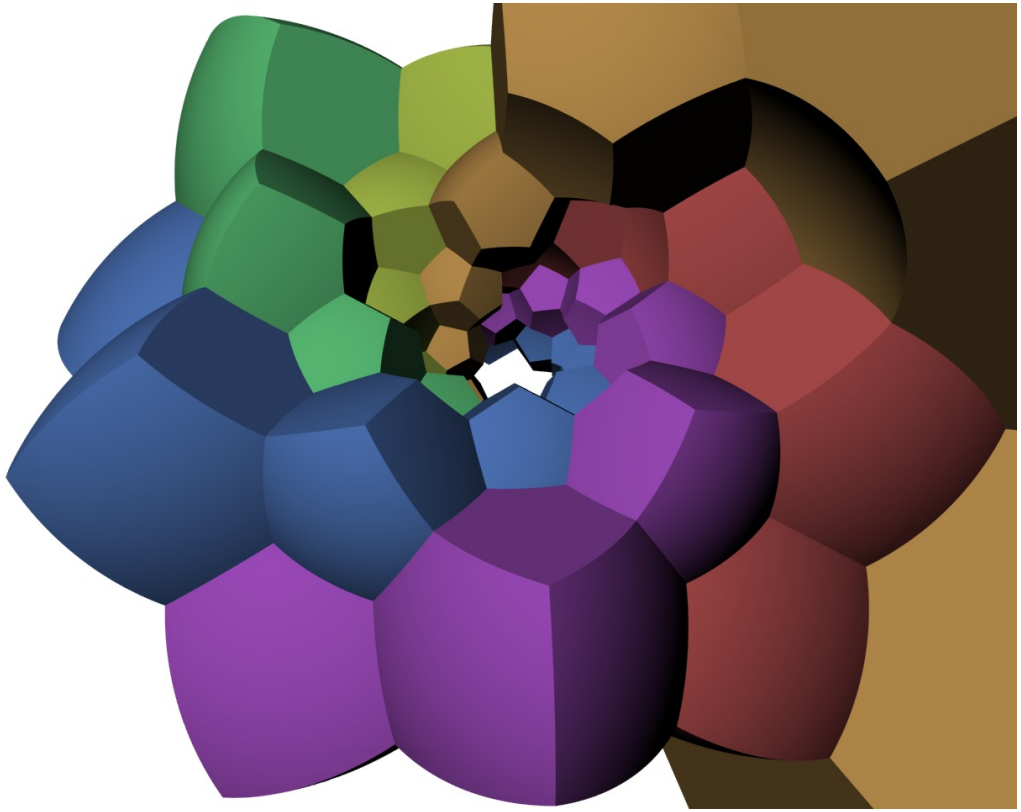

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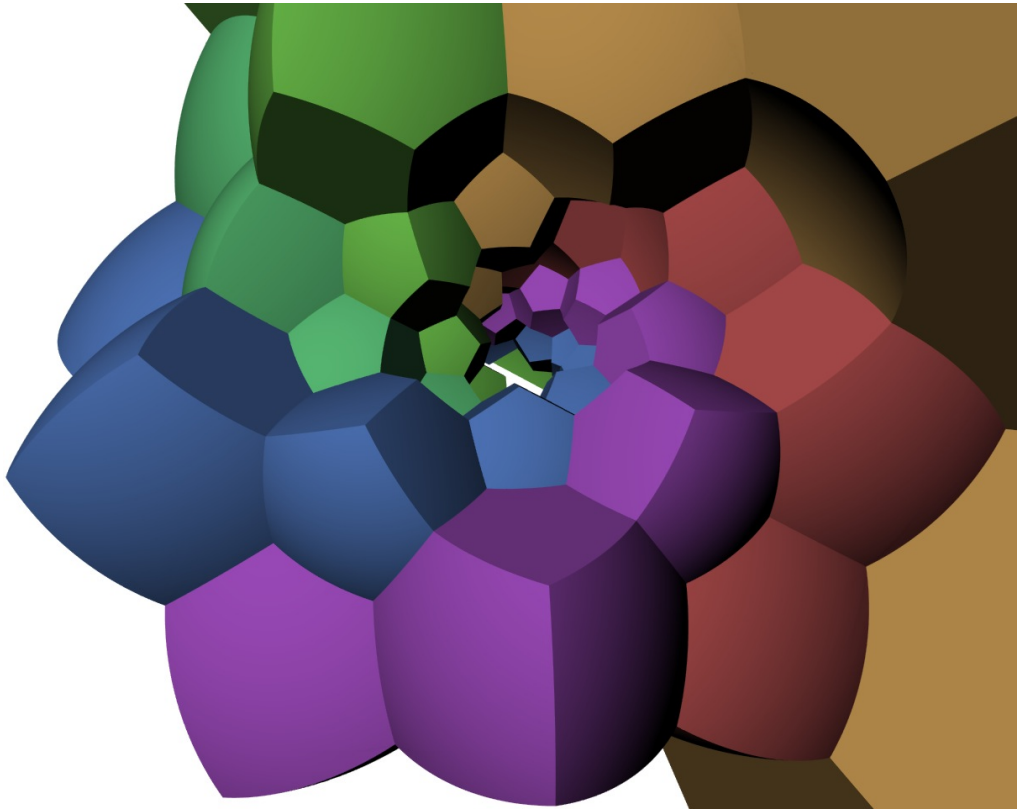

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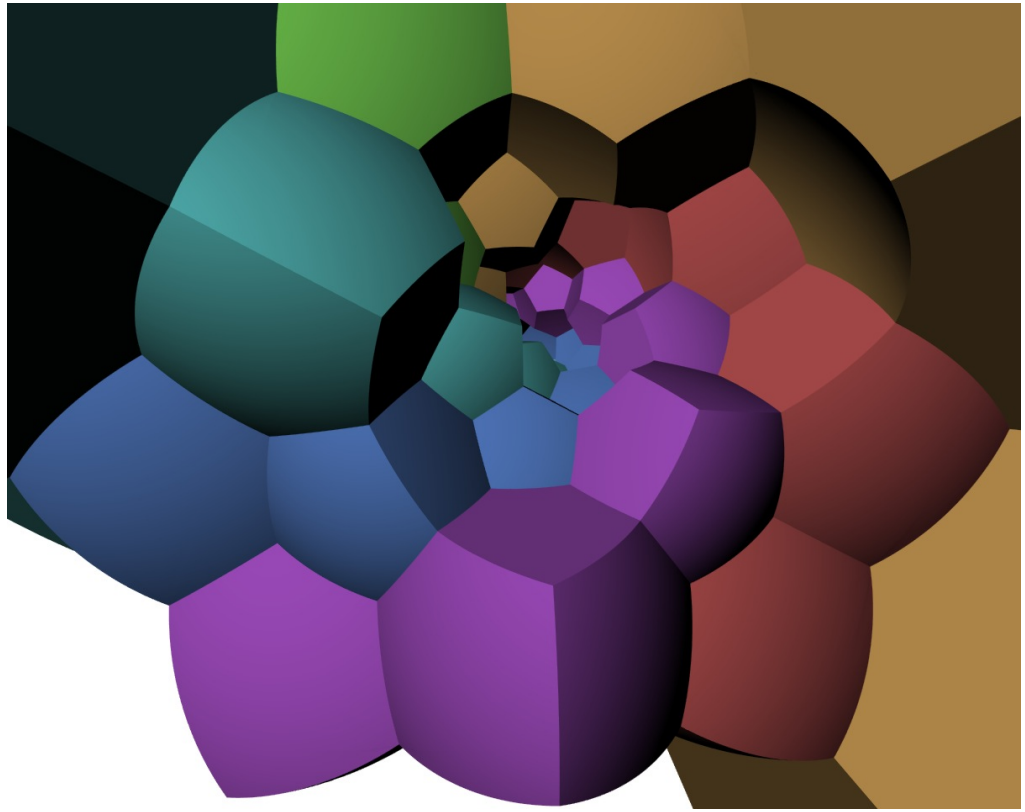

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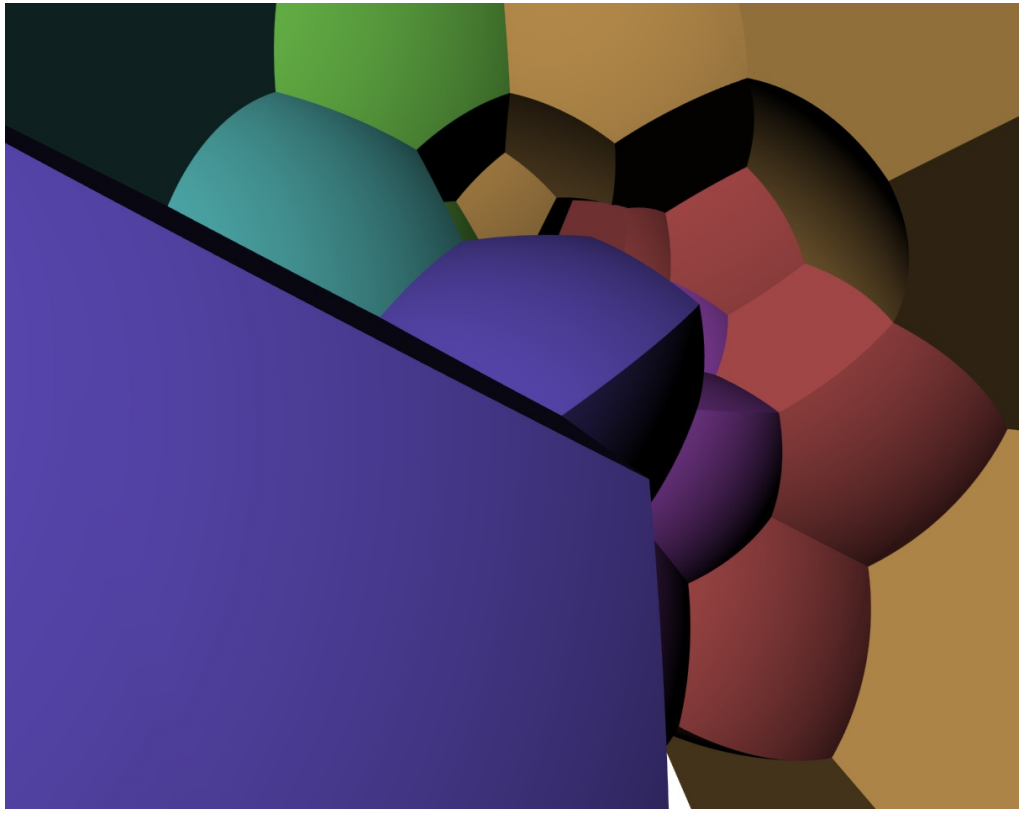

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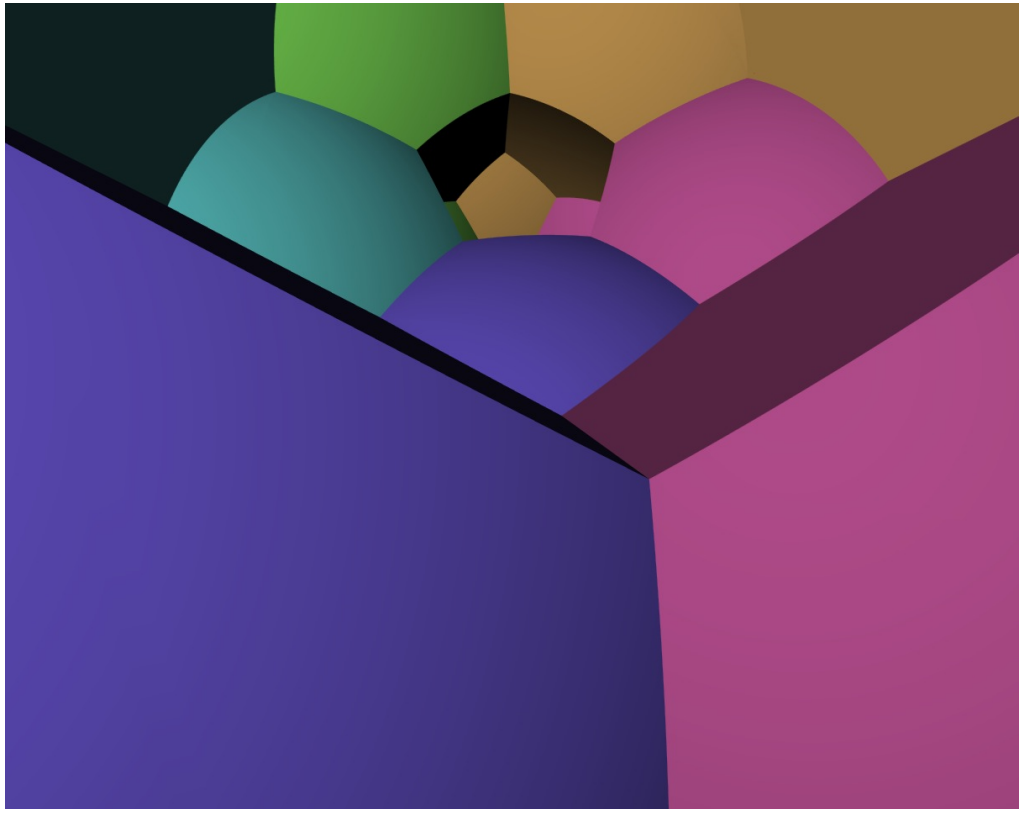

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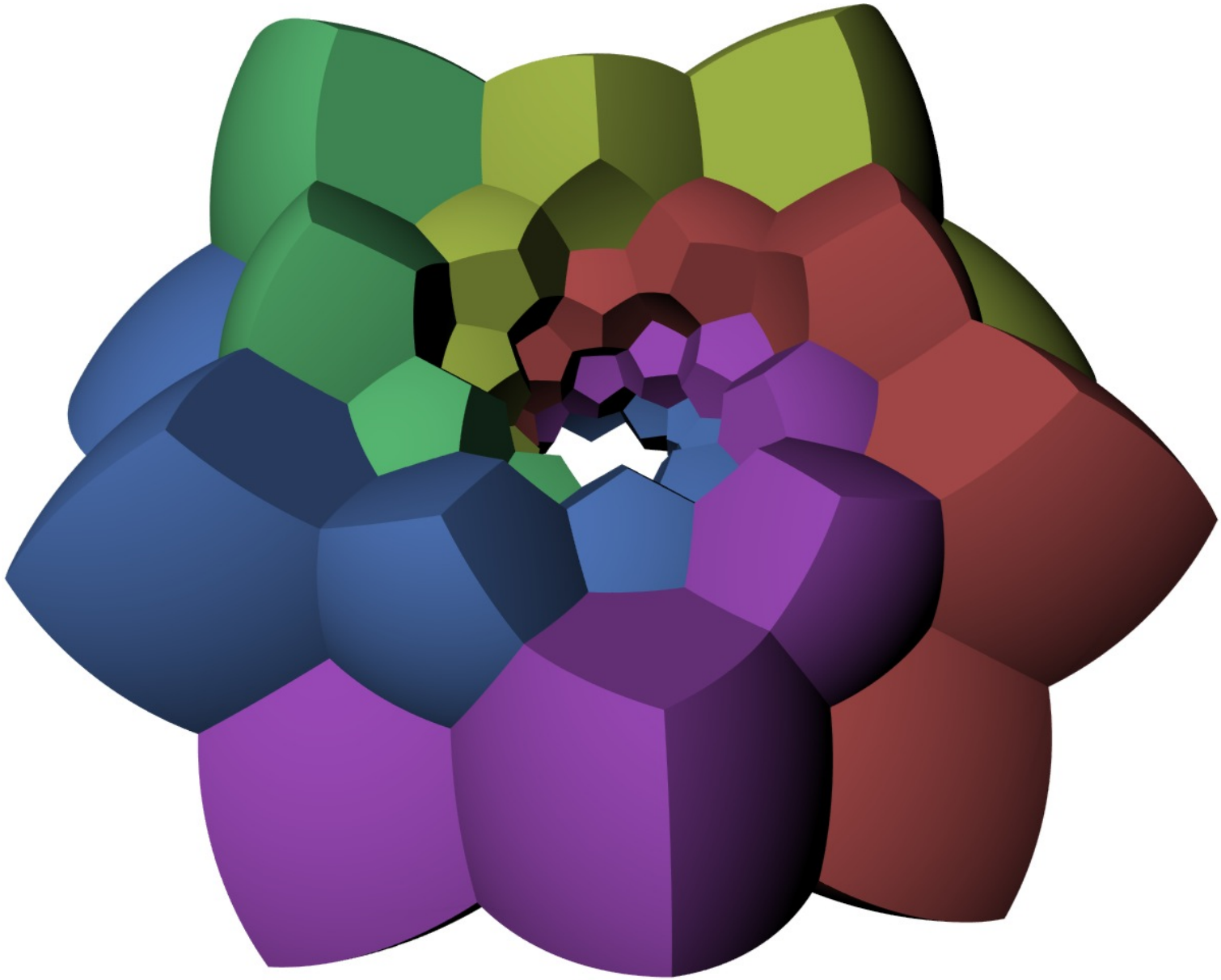

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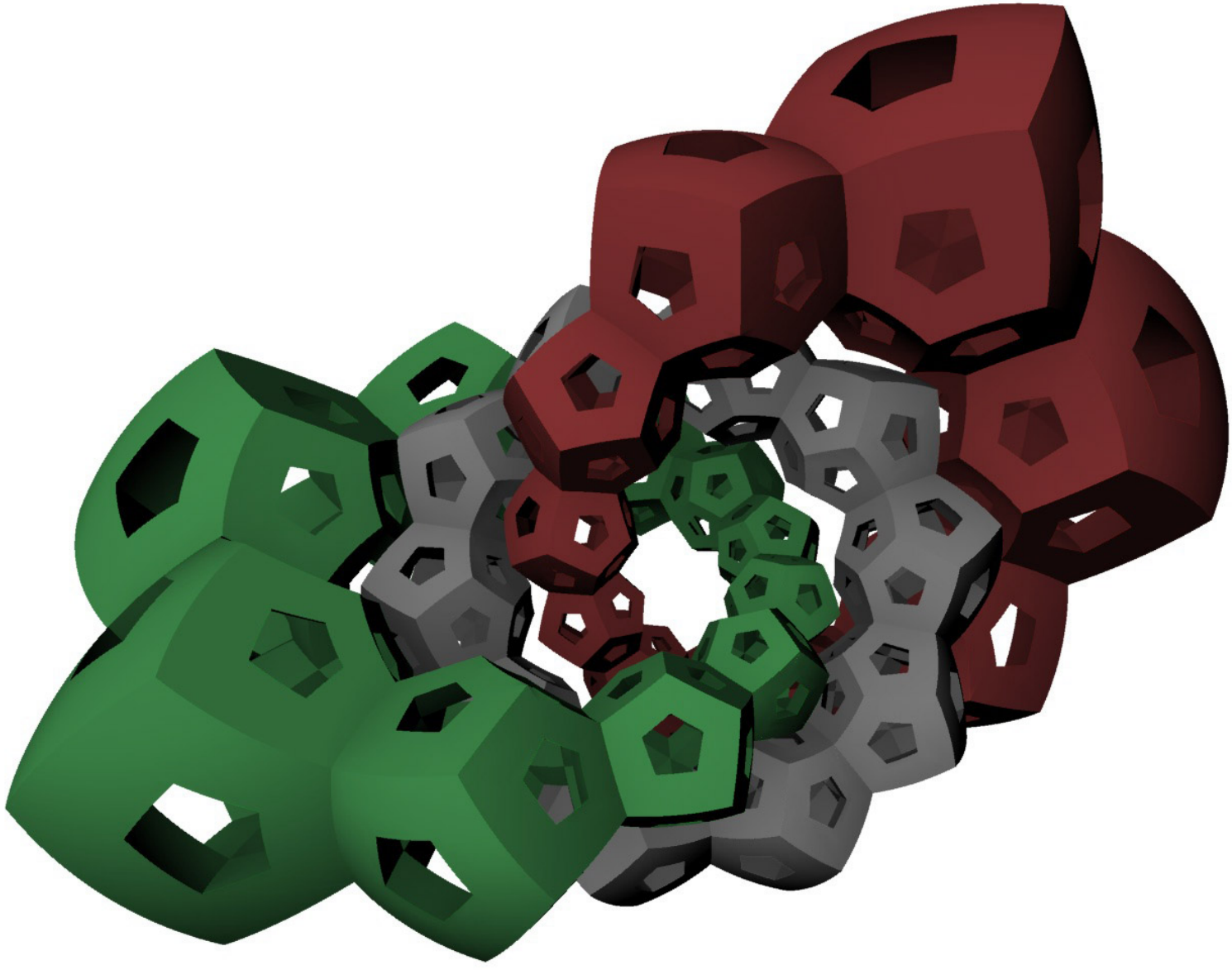
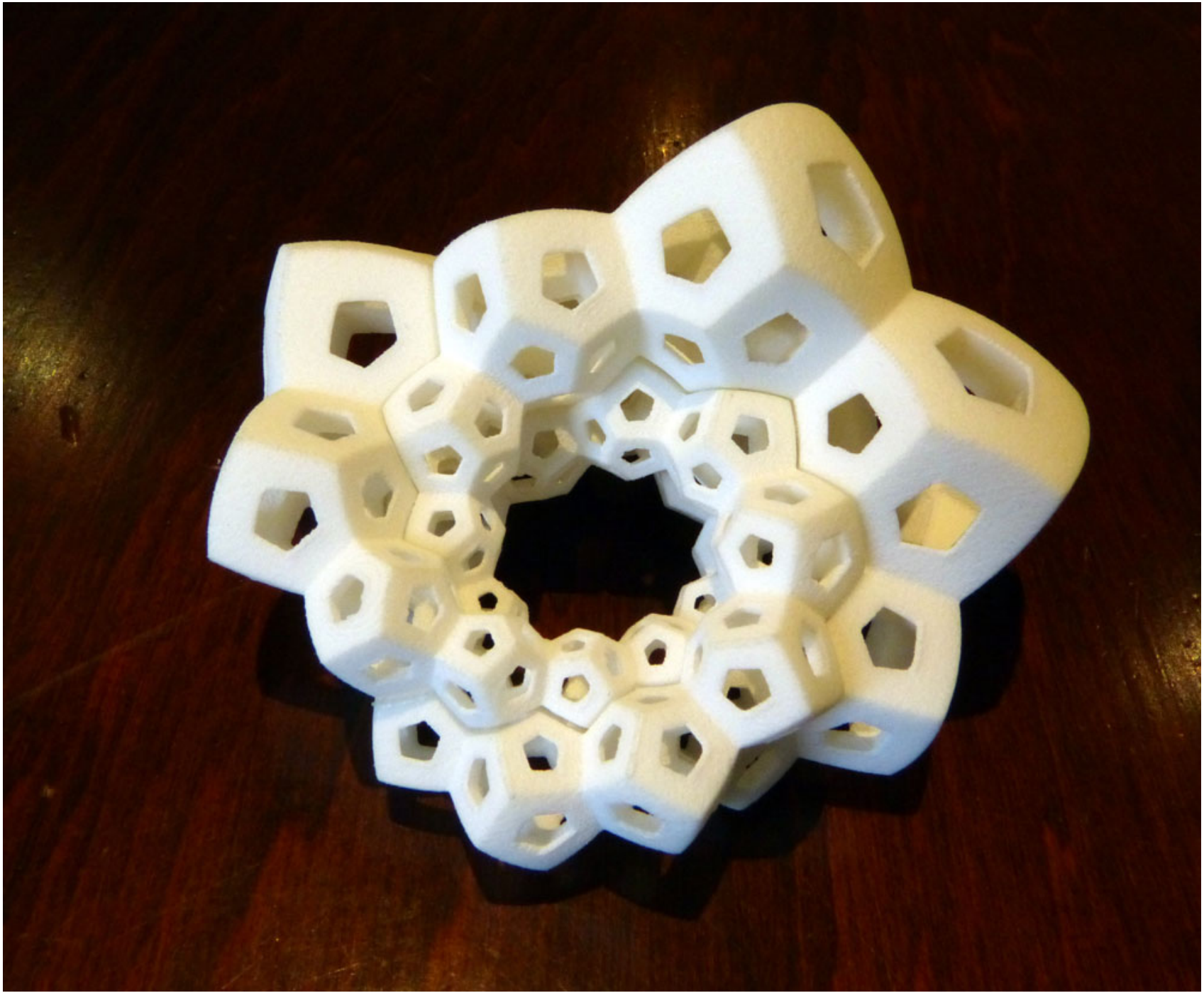
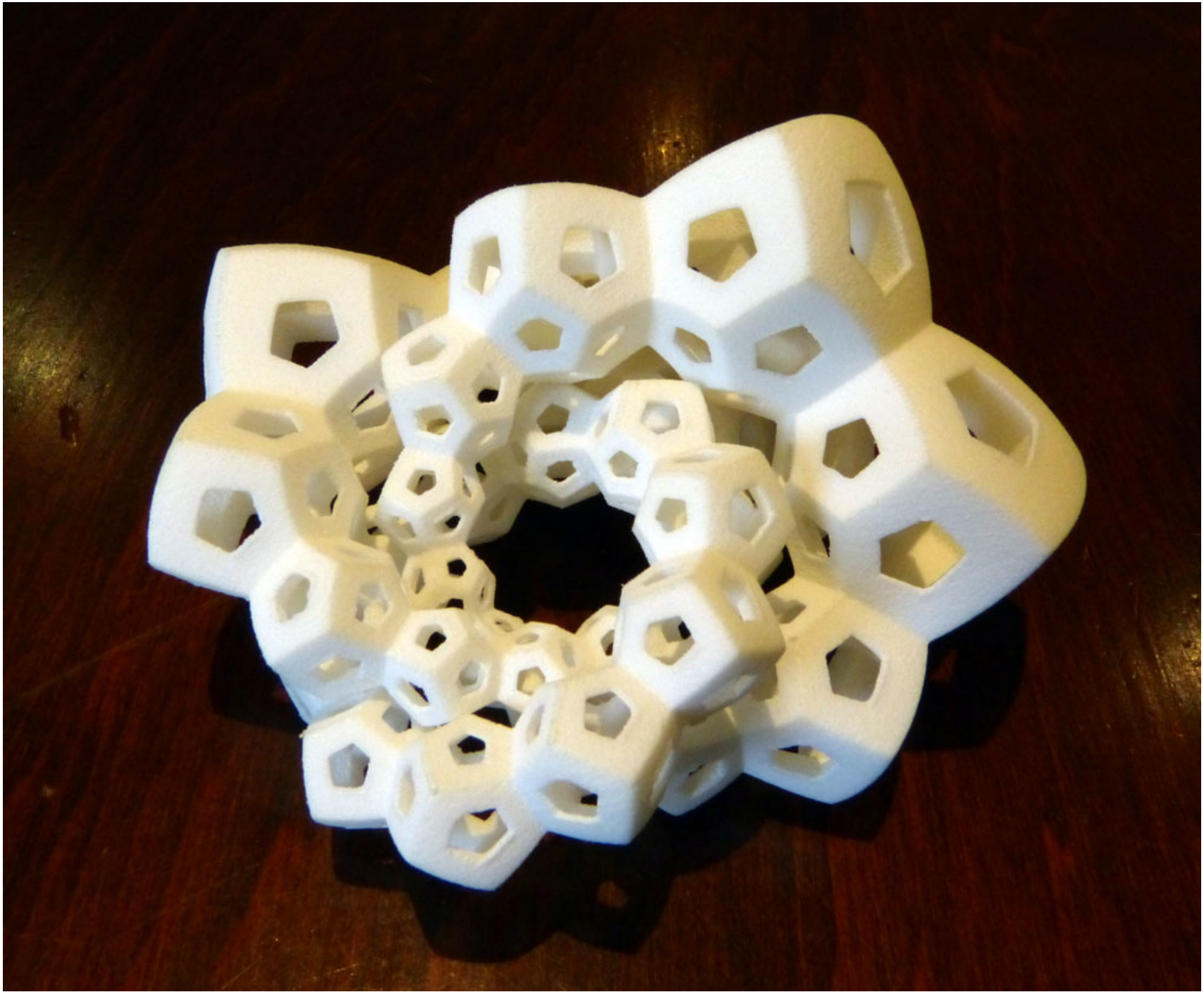
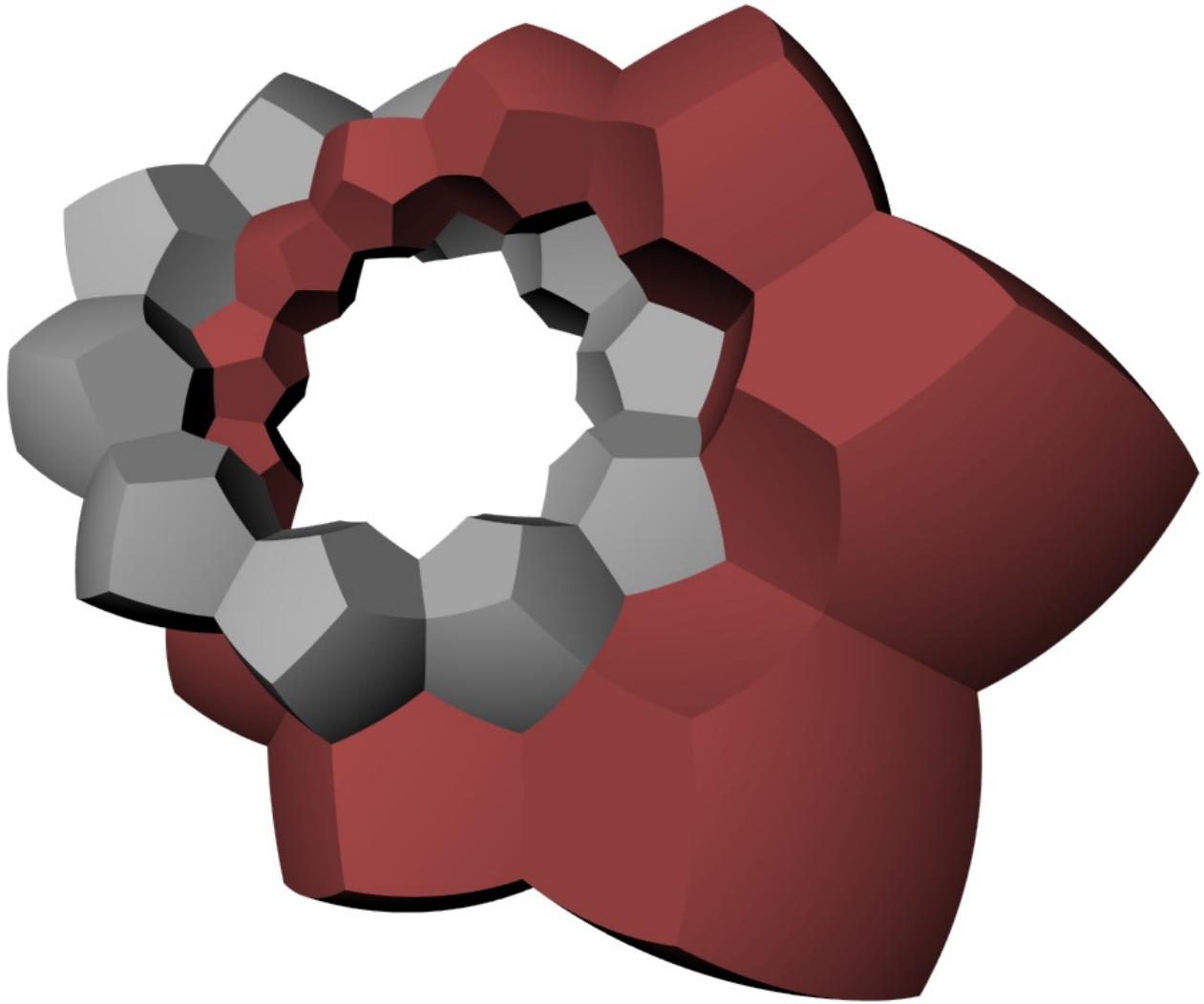






To print all five we use a trick...



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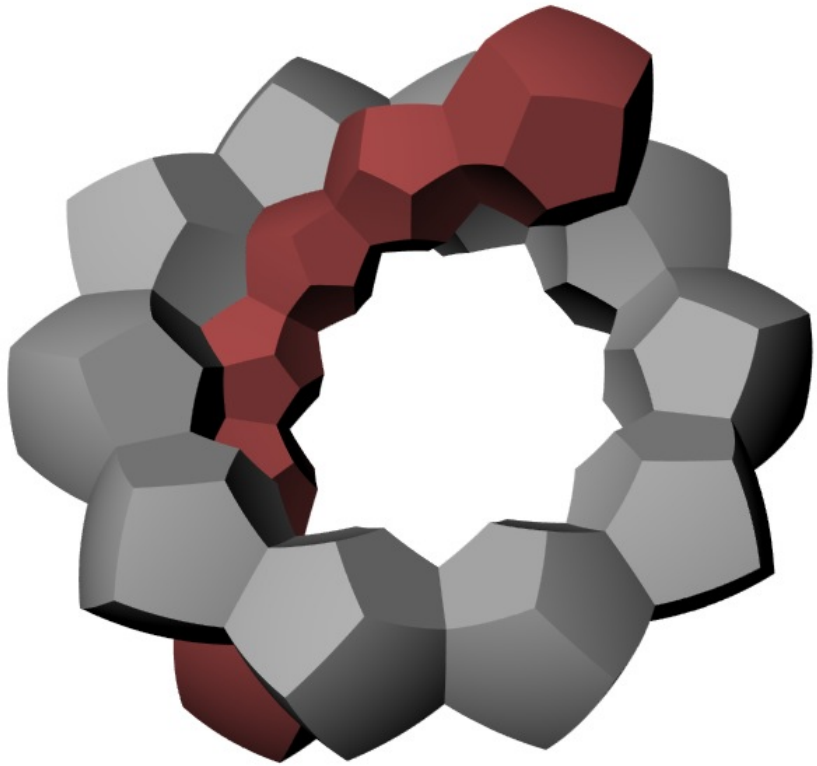

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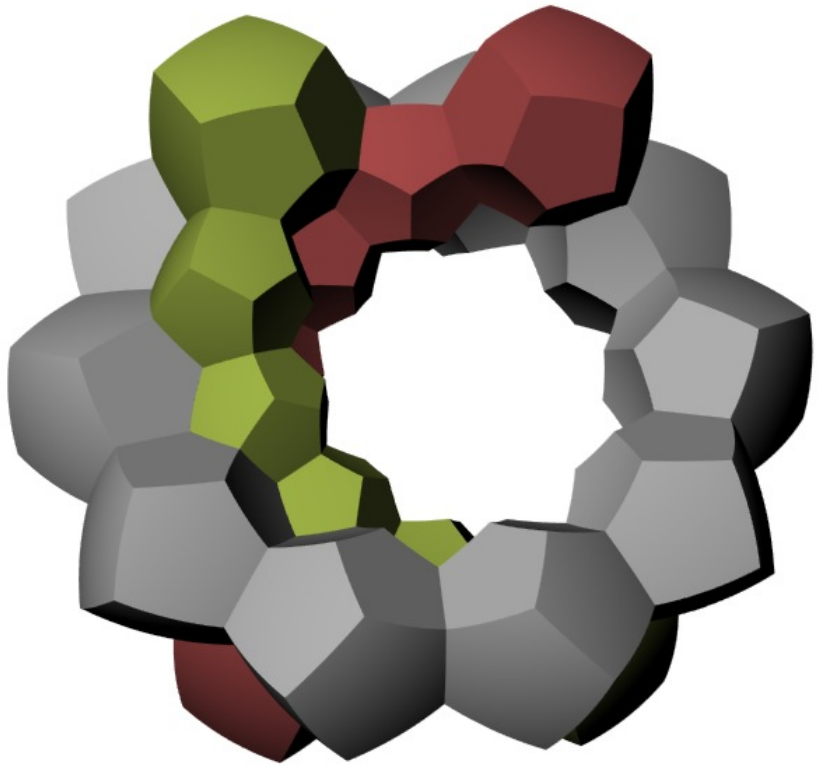

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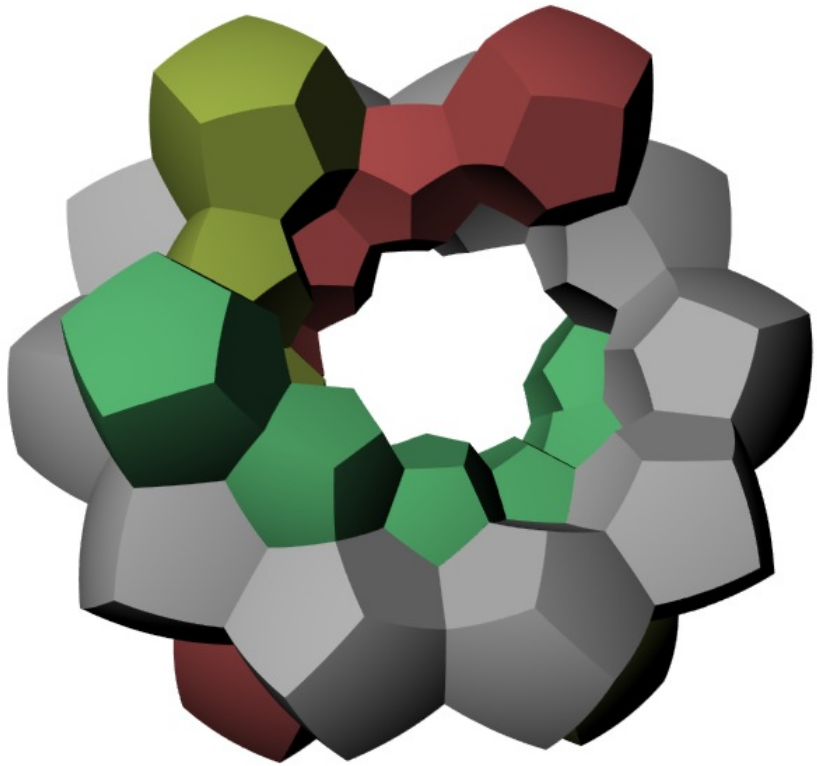

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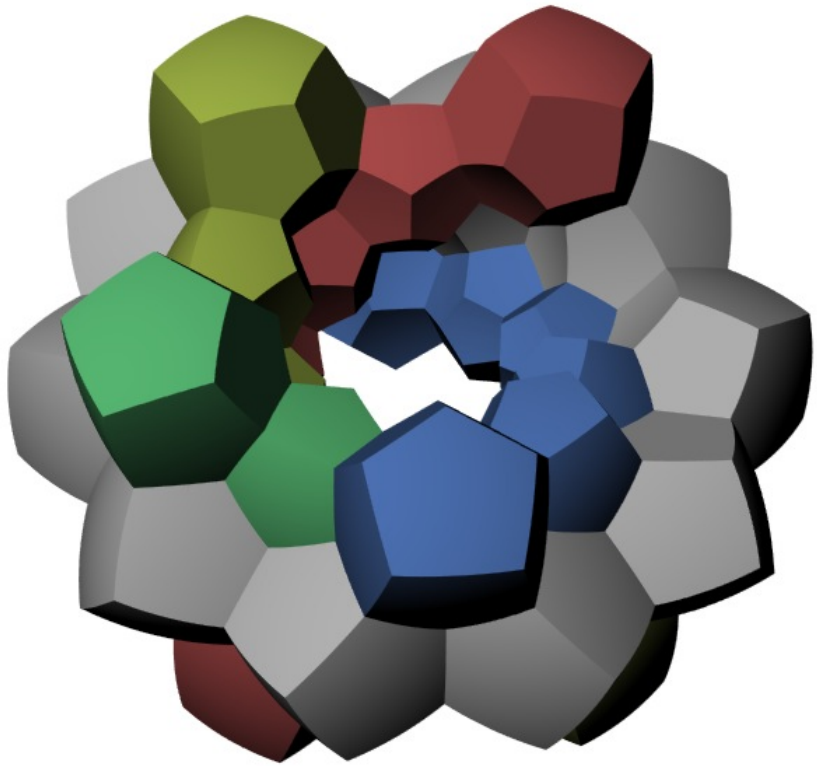

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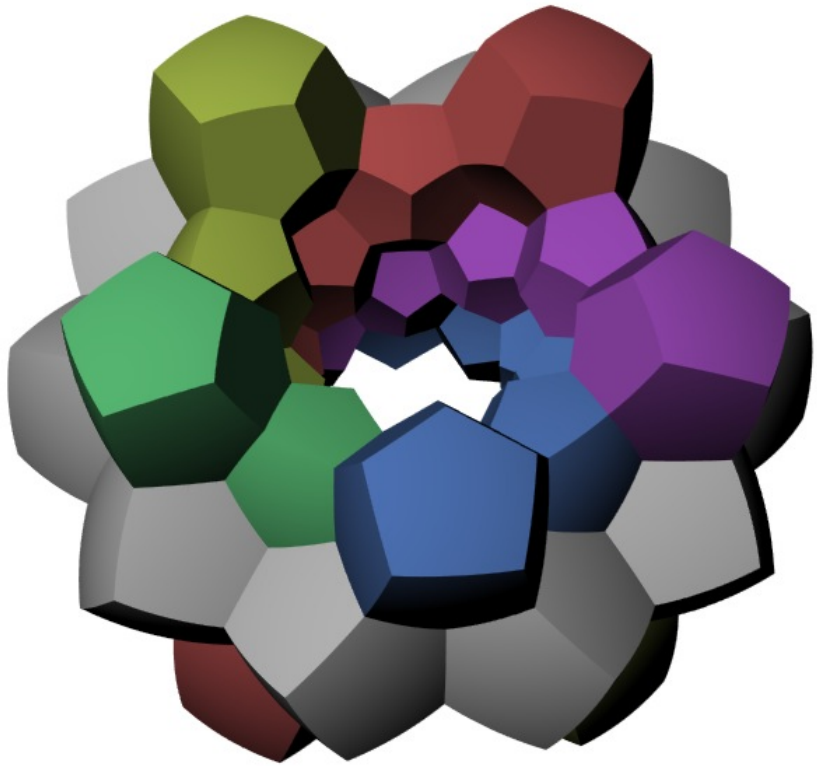

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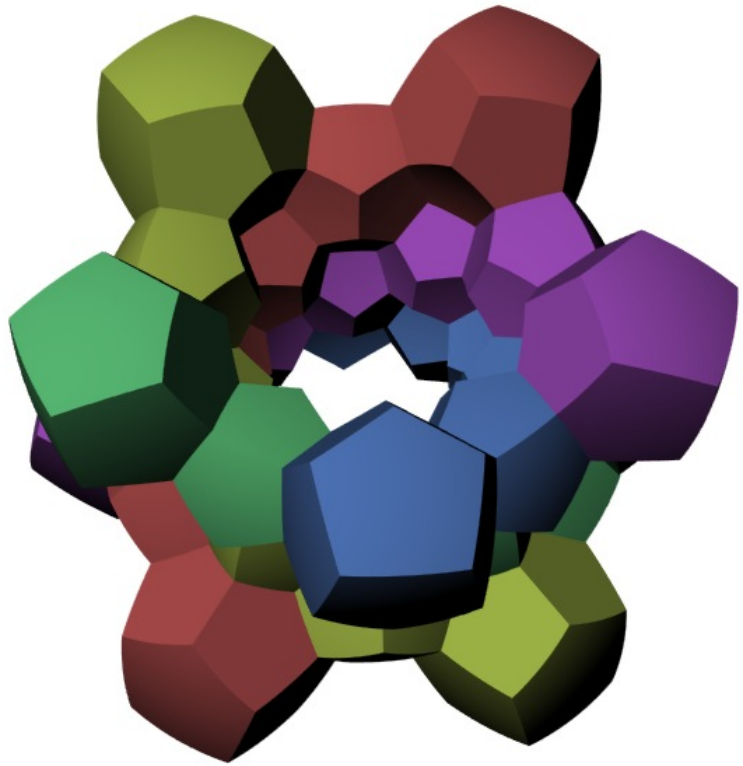

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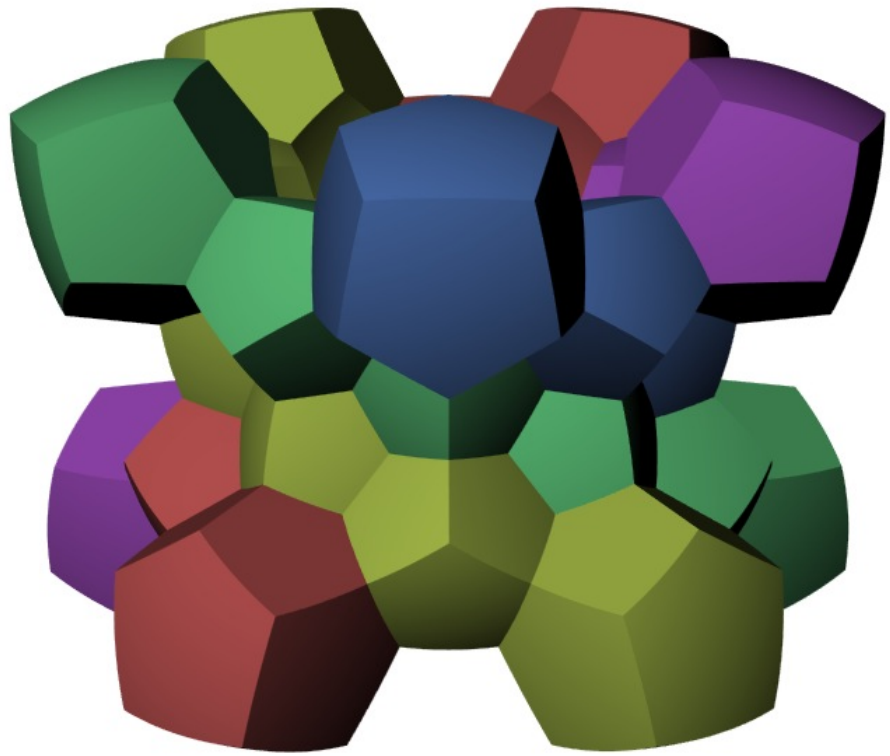
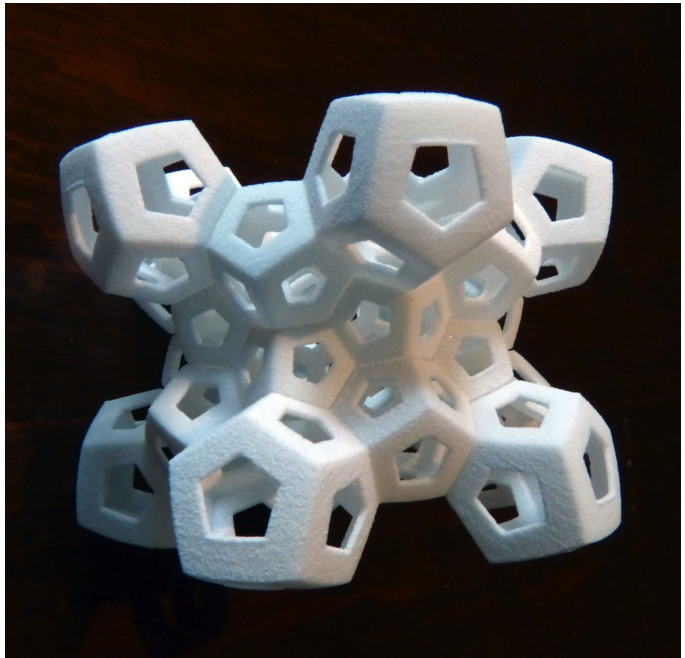
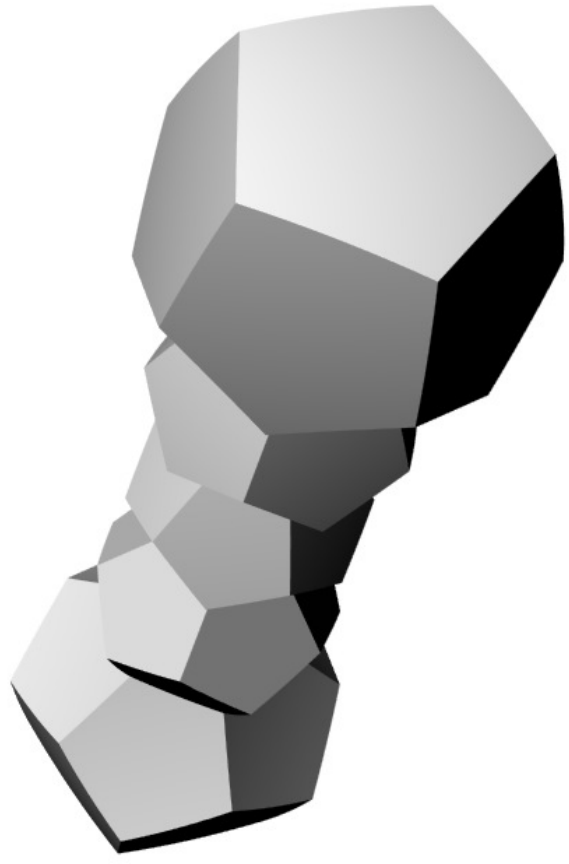


Dc30 Ring puzzle



Another decomposition, with even shorter ribs.



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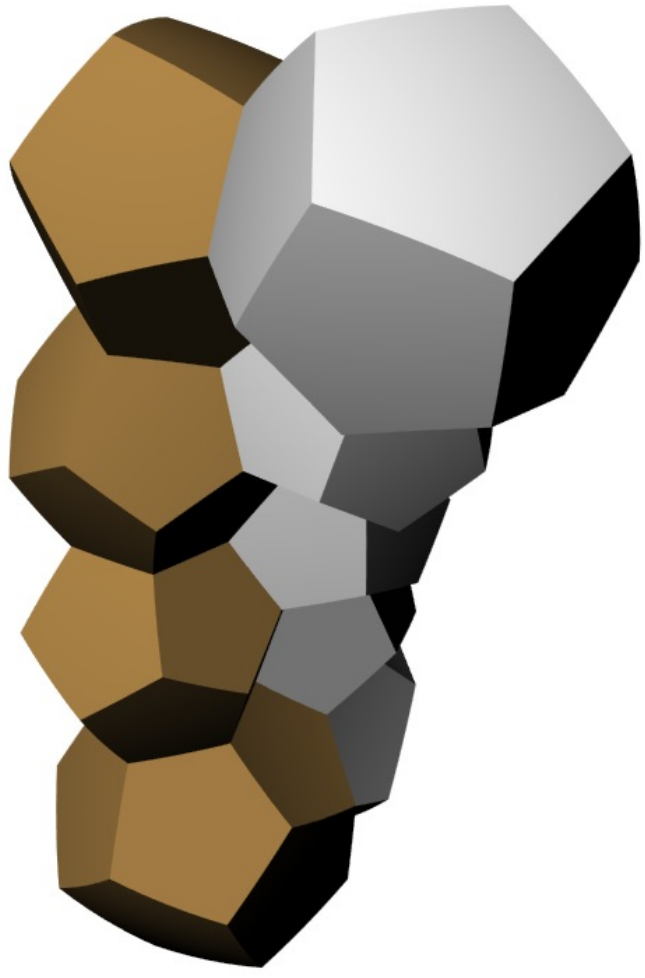

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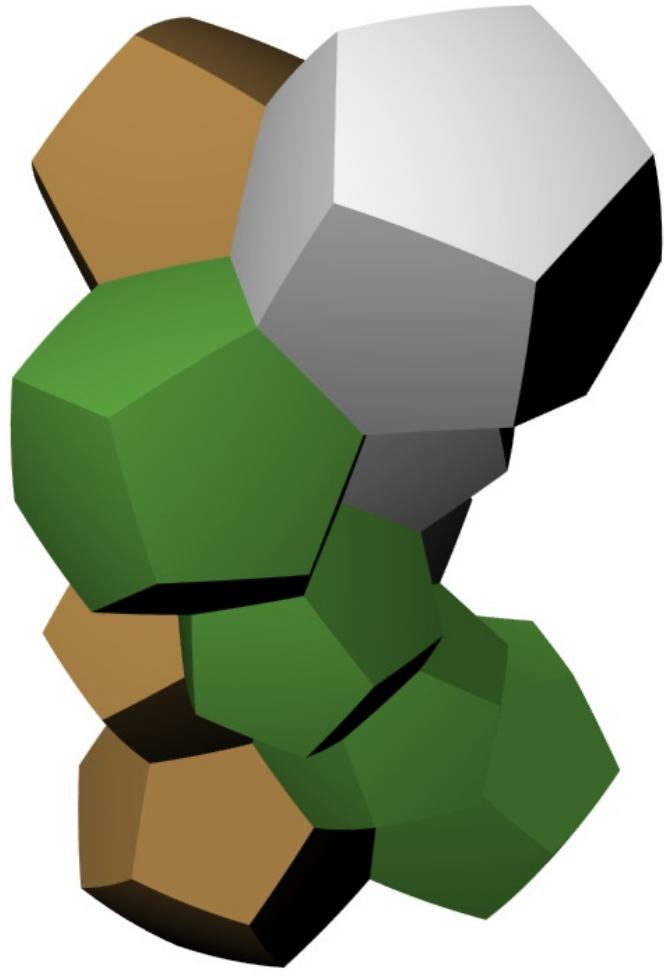

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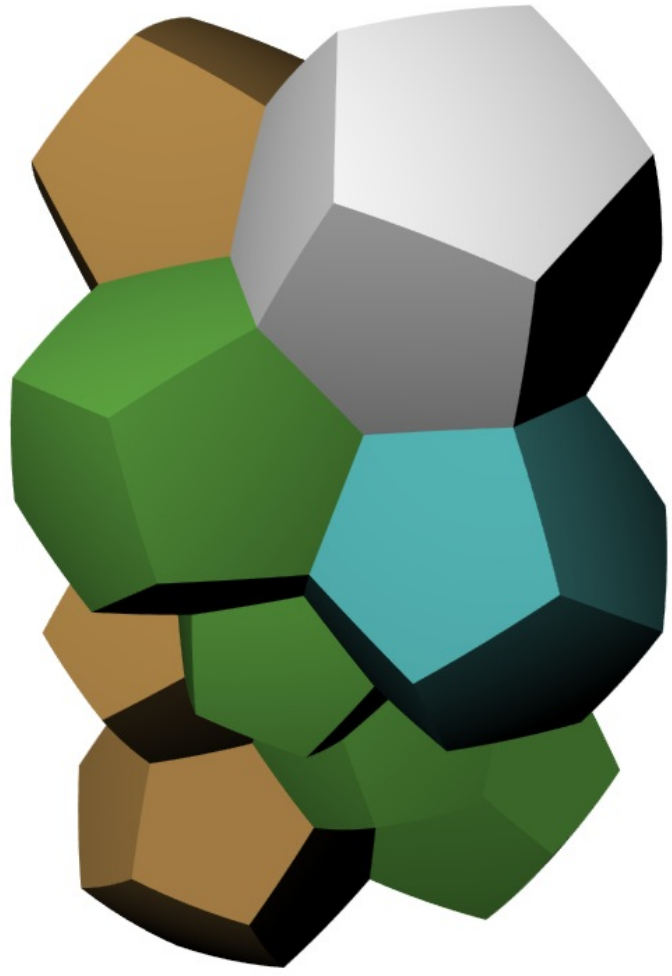

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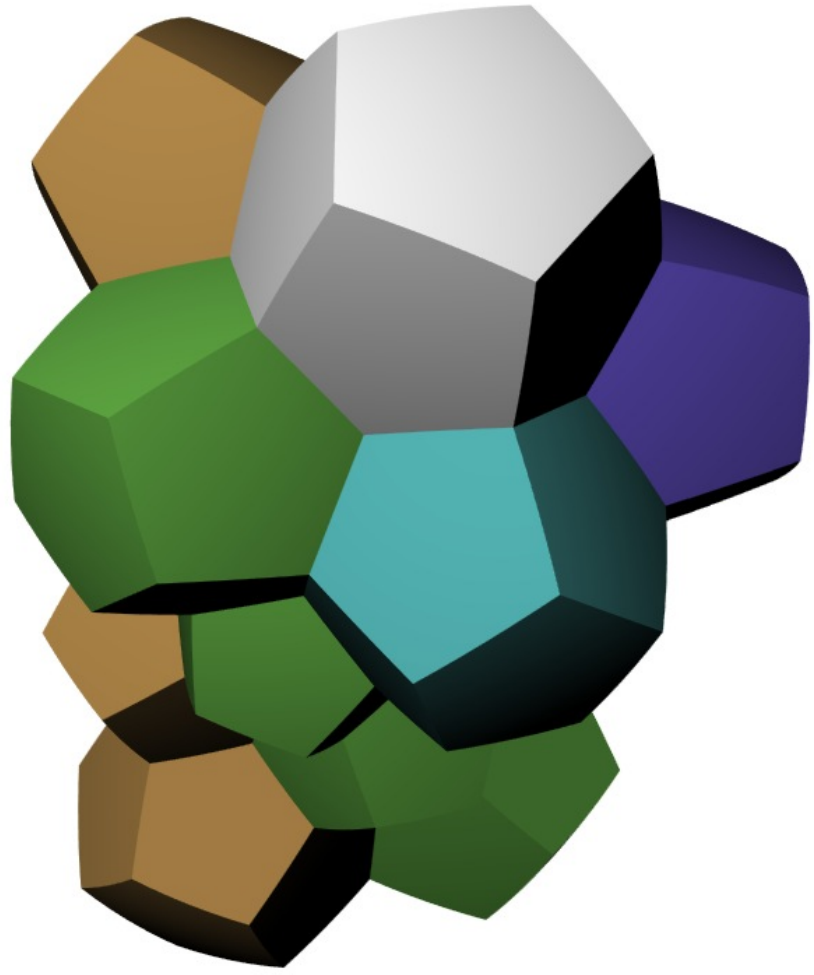

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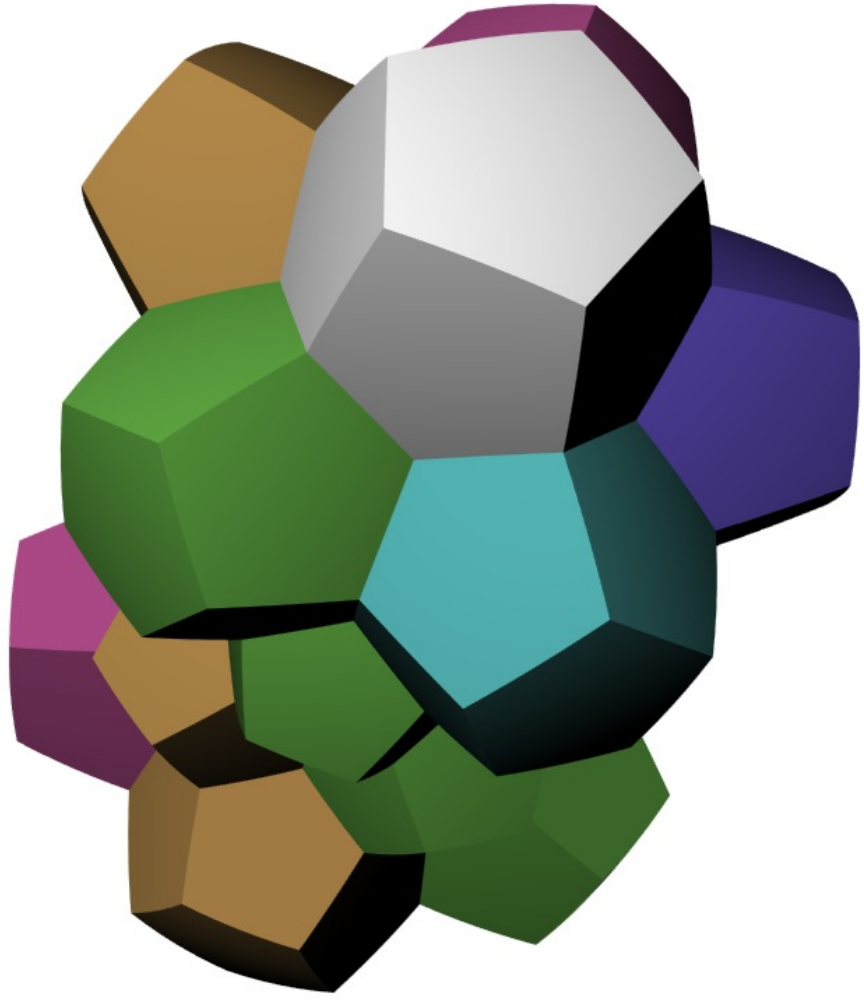

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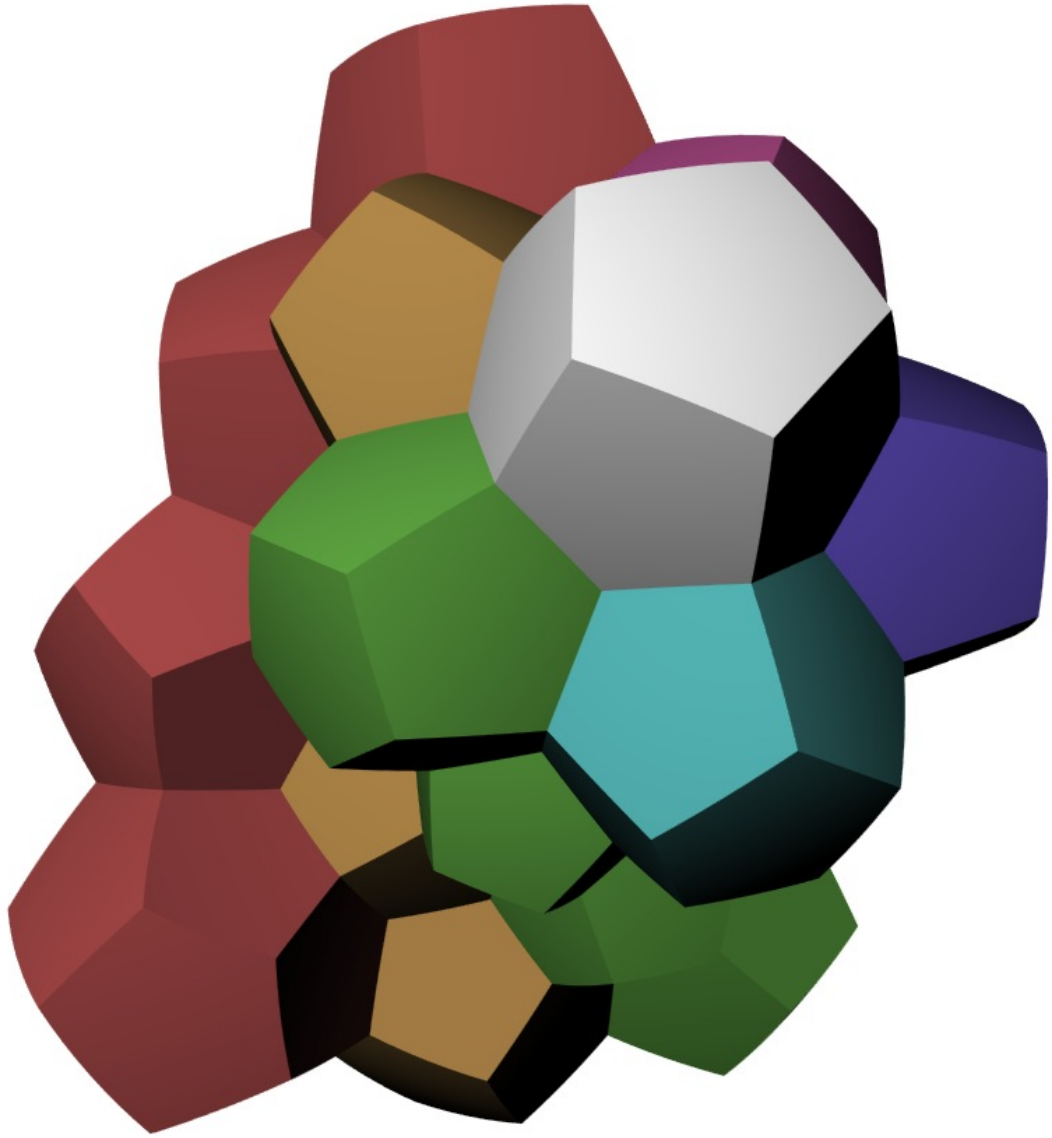

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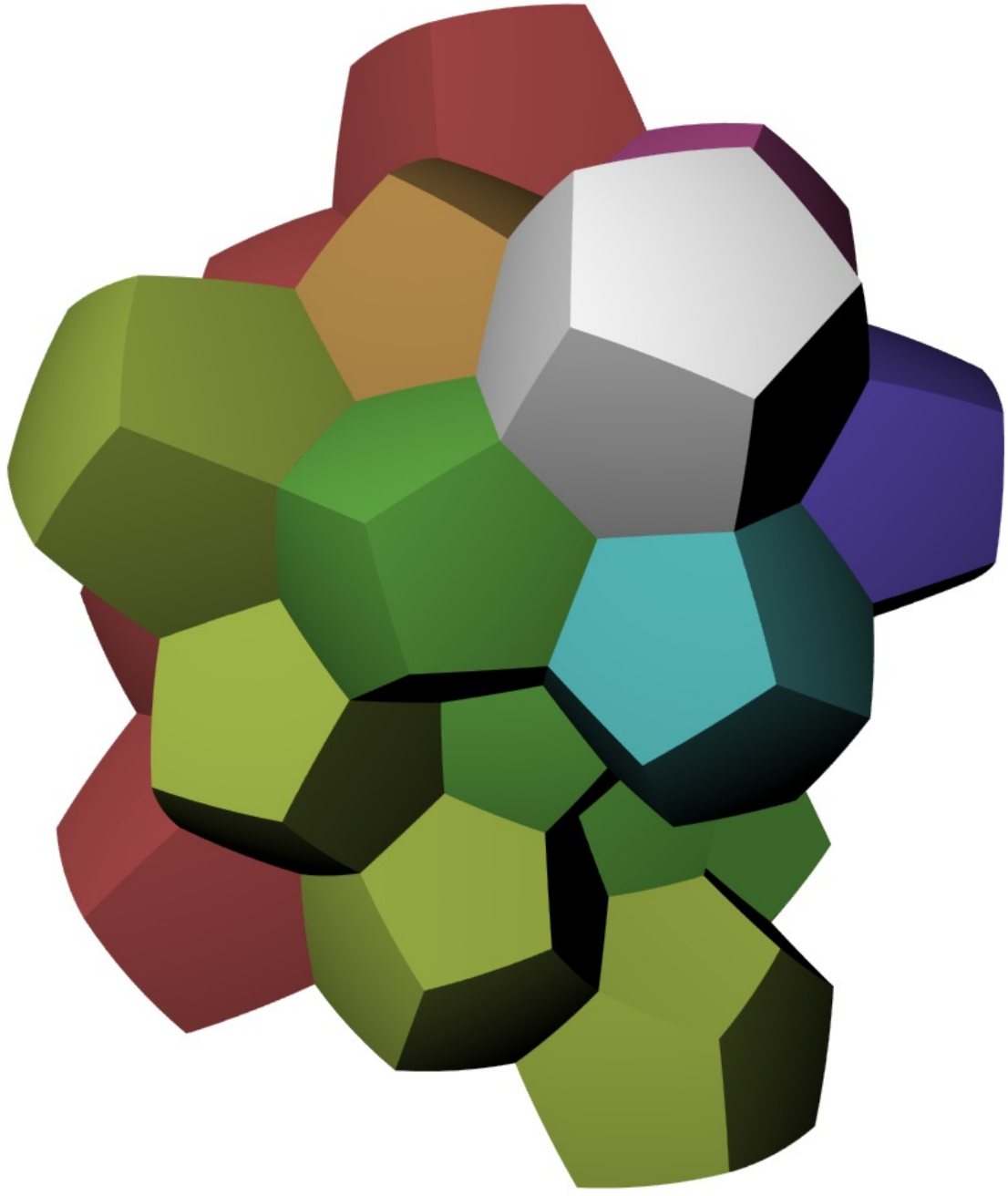

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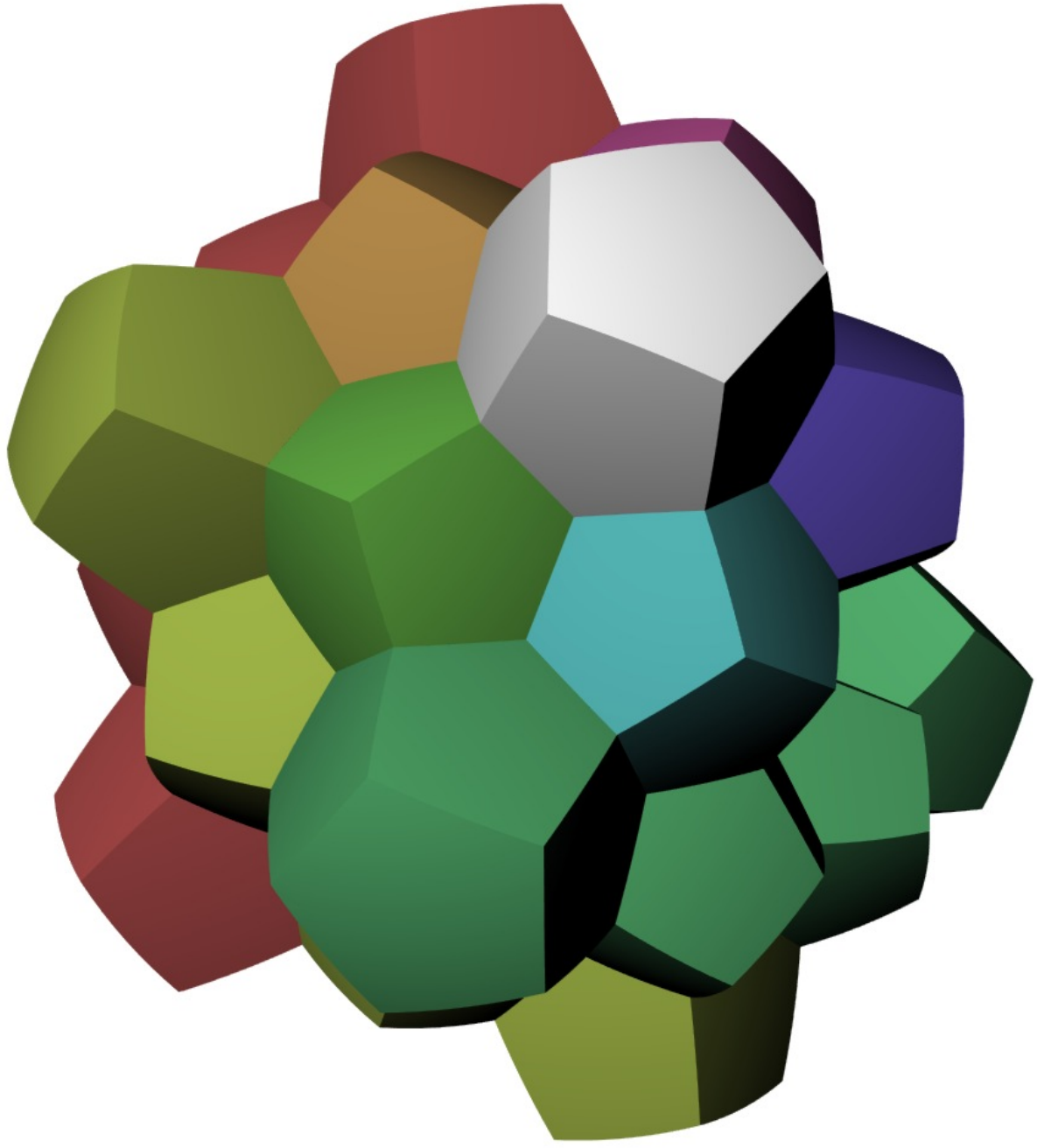

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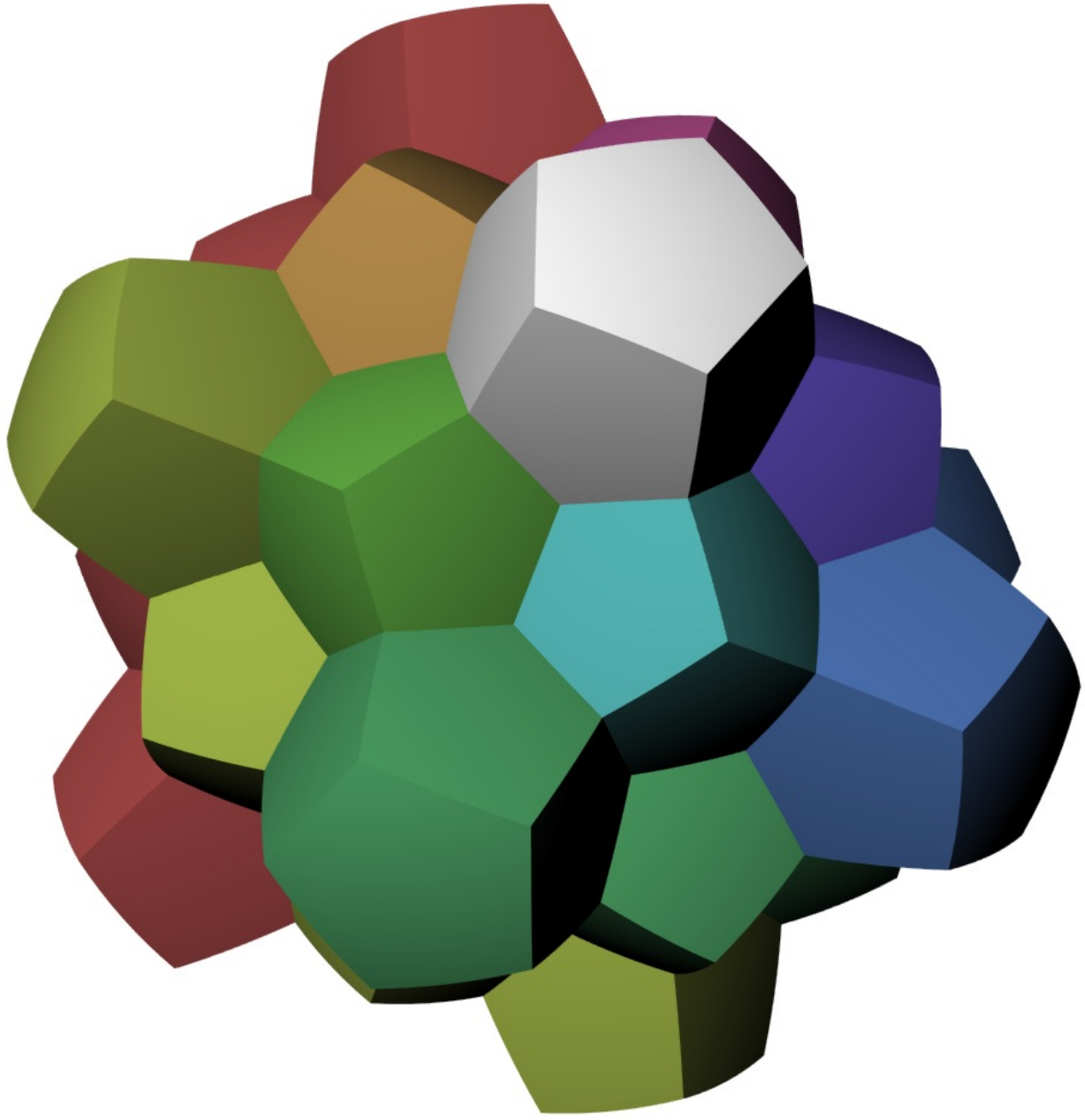

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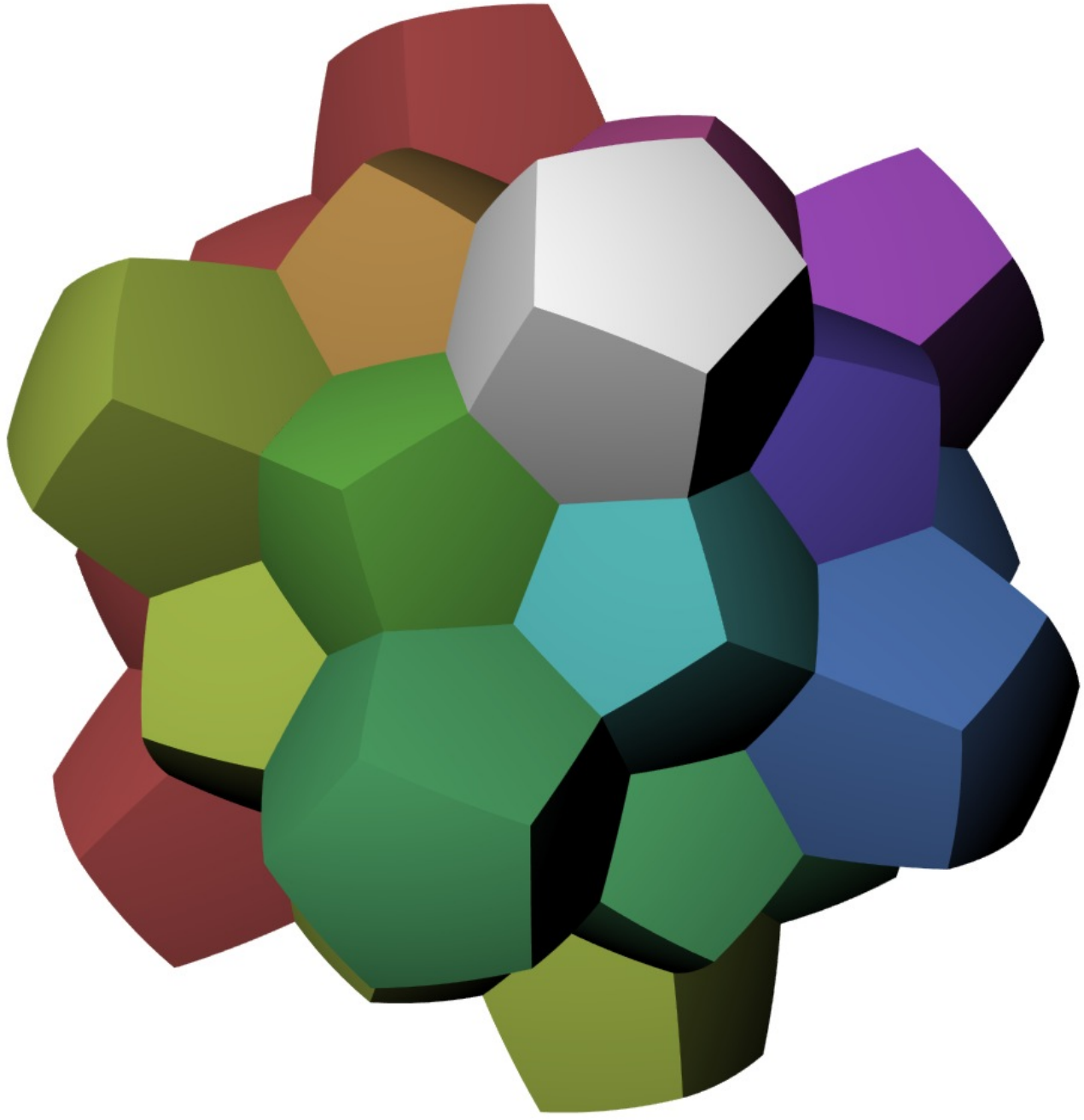
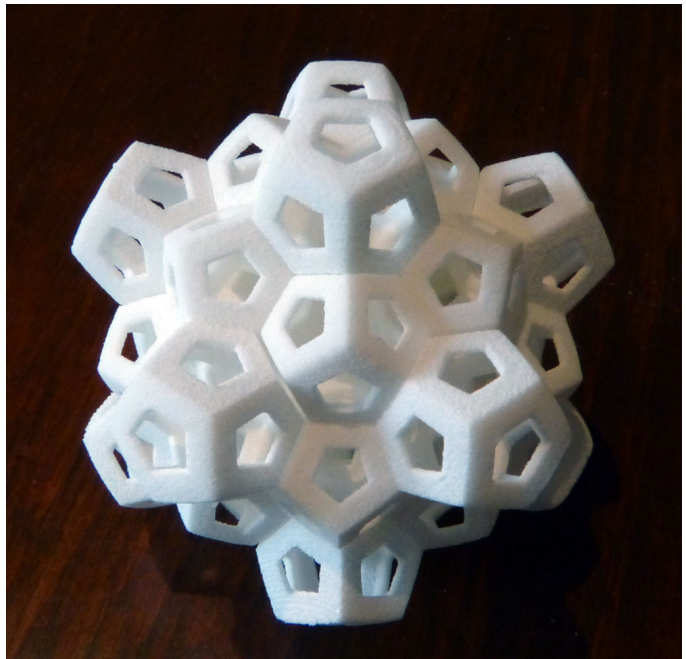
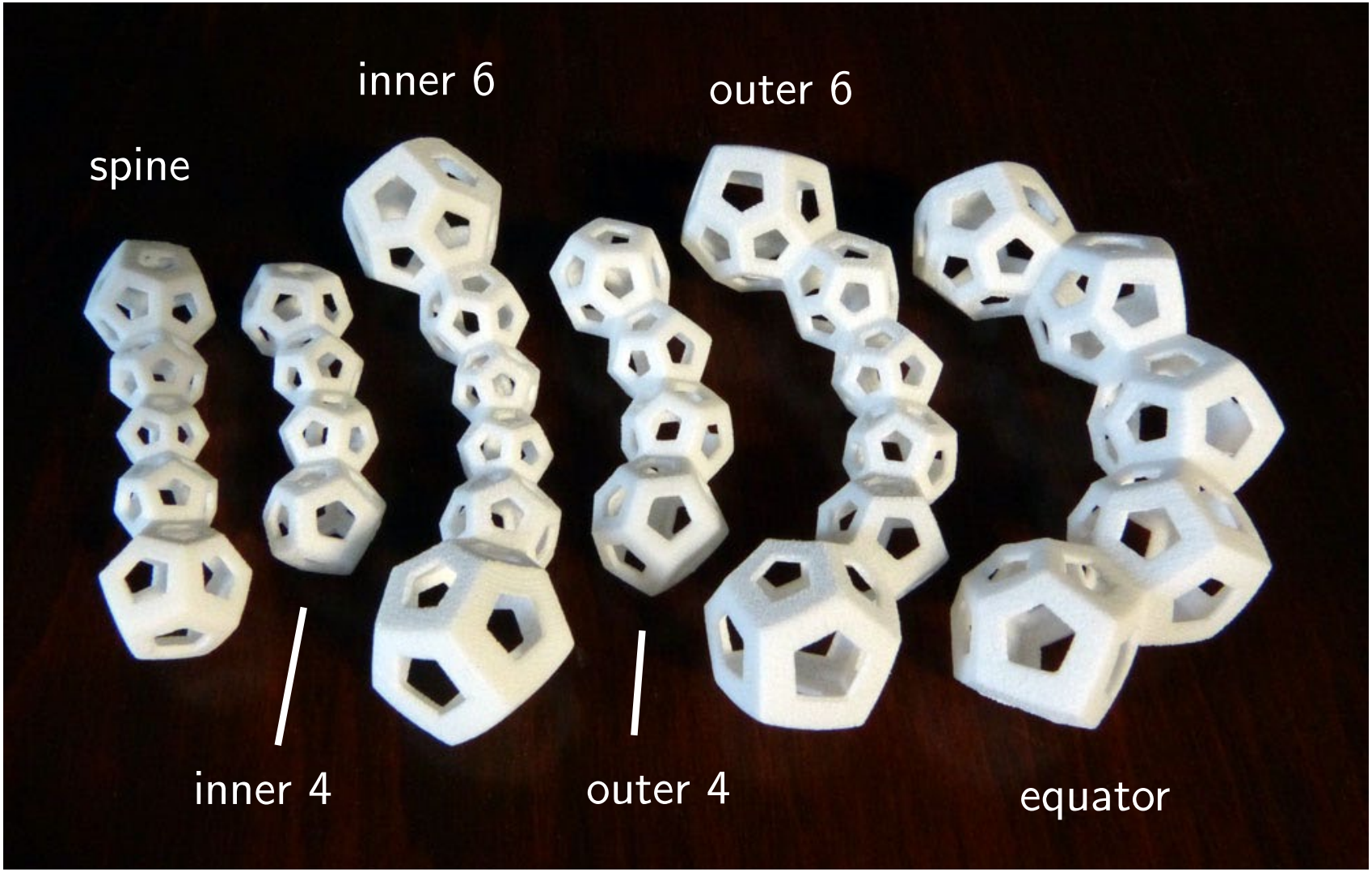


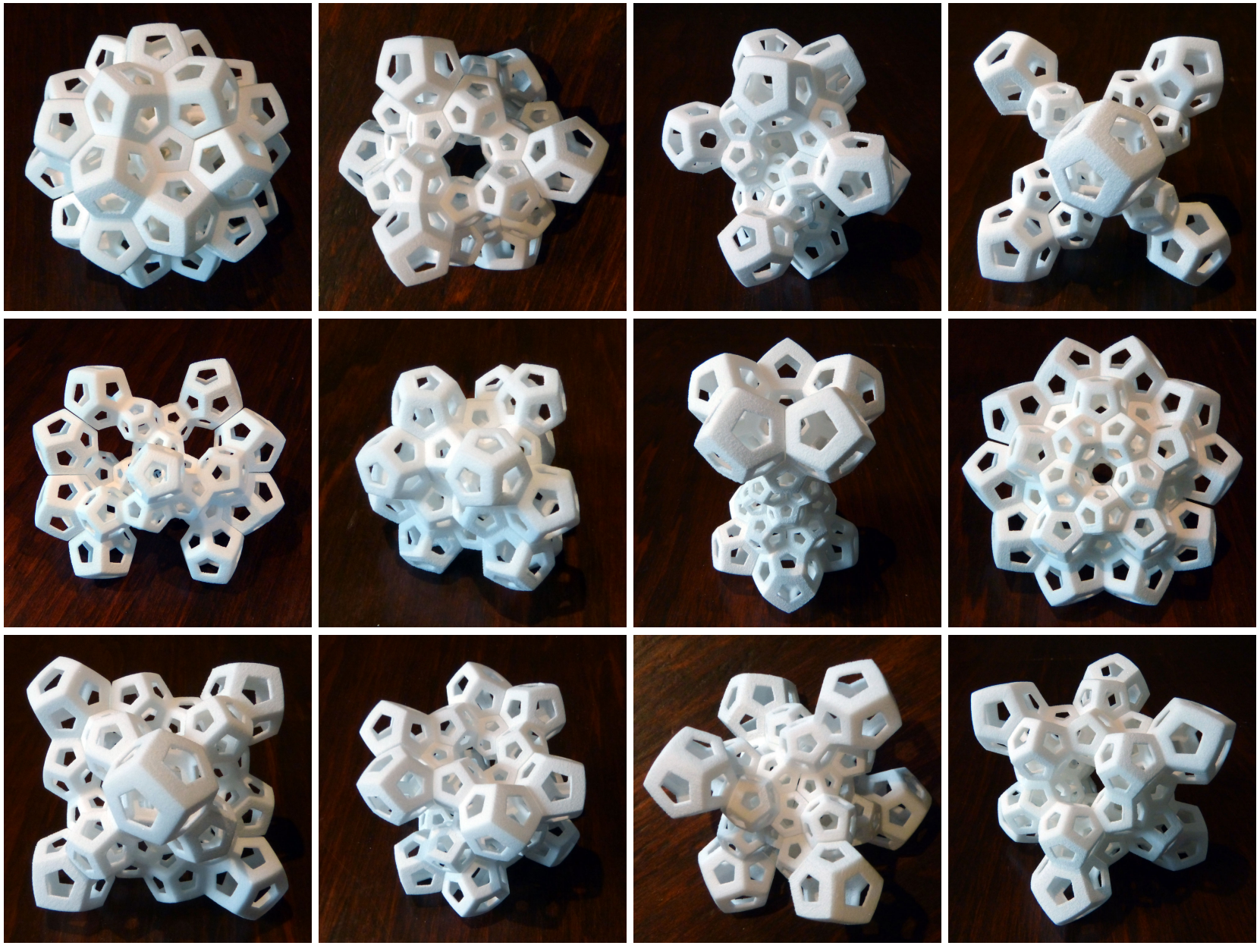
Dc45 Meteor puzzle



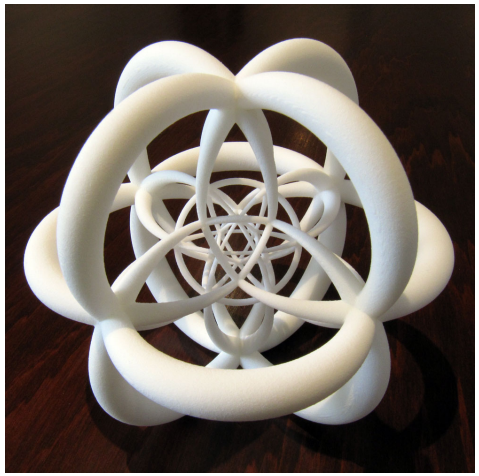
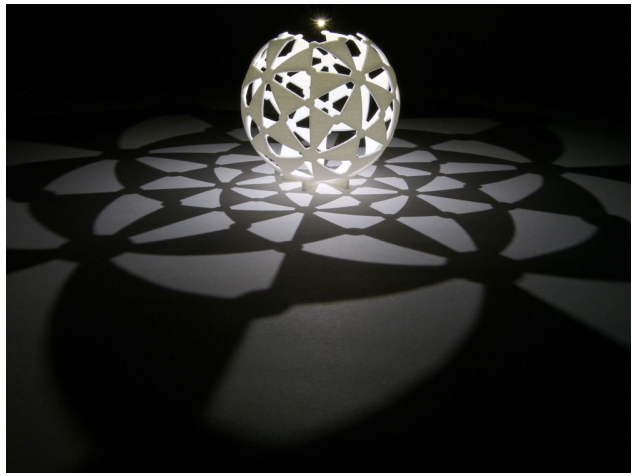
Six kinds of ribs



These make many puzzles, which we collectively call Quintessence.



Thanks!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