Resonant excitation of trapped equatorial waves. Part 2. Wave-Wave-Mean flow Interactions

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Plan:

- 2-layer equatorial dynamics: model and linear wave spectrum.
- The main ideas of the approach and a reminder of previous results.
- Barotropic/baroclinic wave resonance in the presence of the zonal flow.
- Nonlinear saturation in the case of barotropic - baroclinic- mean resonance.
- Effects of spatial modulation.

1 2-layer Equatorial Rotating Shallow Water model and its linear spectrum

1.1 Equations of motion

Momentum and mass conservation equations in each layer:

$$\partial_t \mathbf{u}_i + \mathbf{u}_i \cdot \nabla \mathbf{u}_i + \beta y \hat{\mathbf{z}} \times \mathbf{u}_i + \frac{1}{\rho_i} \nabla \pi_i = 0\,, i = 1, 2; \tag{1}$$

$$\partial_t h_i + \nabla \cdot (\mathbf{u}_i h_i) = 0\,, i = 1, 2, \tag{2}$$

$\mathbf{u}_i = (u_i(x, y, t),\, v_i(x, y, t))$ - velocities, ρ_i - densities of the layers, close to each other,

$$\pi_2 = \pi_1 - \rho_1 g' h_1 \quad g' = g(\rho_2 - \rho_1)/\rho_1\,, \tag{3}$$

B.c.: rigid lid and flat bottom.

Barotropic and the baroclinic velocities:

$$\mathbf{u}_{bt} = \frac{h_1 \mathbf{u}_1 + h_2 \mathbf{u}_2}{H}, \ \mathbf{u}_{bc} = \mathbf{u}_1 - \mathbf{u}_2. \tag{4}$$

Rigid lid $\Rightarrow \nabla \cdot \mathbf{u}_{bt} = 0 \rightarrow$ barotropic streamfunction ψ.

Equations of motion:

$$\begin{aligned} \nabla^2 \psi_t + \psi_x &= \epsilon \left[-J(\psi, \nabla^2 \psi) - s(\partial_{xx} - \partial_{yy}) \left[(1 + \epsilon q h)(uv) \right] \right. \\ &+ \left. s \partial_{xy} \left[(1 + \epsilon q h)(u^2 - v^2) \right]) \right] \end{aligned} \tag{5}$$

$$\begin{aligned} \mathbf{u}_t + \nabla h + y \hat{\mathbf{z}} \times \mathbf{u} &= \epsilon \left[-J(\psi, \mathbf{u}) + \mathbf{u} \cdot \nabla (\hat{\mathbf{z}} \times \nabla \psi) - q \mathbf{u} \cdot \nabla \mathbf{u} \right. \\ &+ \left. \epsilon s \left(2h \mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{u} \, \mathbf{u} \cdot \nabla h \right) \right], \end{aligned} \tag{6}$$

$$h_t + \nabla \cdot \mathbf{u} = \epsilon \left[-J(\psi, h) - q \nabla \cdot (\mathbf{u} h) + \epsilon s \nabla \cdot \left(h^2 \mathbf{u} \right) \right]. \tag{7}$$

1.2 Parameters and scaling

$$q = \frac{H - 2H_1}{H}, \quad s = \frac{H_s}{H}, \quad \epsilon = \frac{\Delta H}{H_s}, \tag{8}$$

ΔH denotes a typical variation of the interface, $H_s = \frac{H_1(H-H_1)}{H}$. The following scaling is used to non-dimensionalize the model:

$$L = \frac{(g' H_s)^{\frac{1}{4}}}{\sqrt{\beta}}; \; T = \frac{1}{\beta L}; \quad U = \frac{g' \Delta H}{\beta L^2}. \tag{9}$$

1.3 Linear wave spectrum

Barotropic Rossby waves - propagate at any angle with respect to the equator:

$$\tilde{\psi}_0 = A_\psi e^{i(\theta + ly)} + c.c.;\ \theta = kx - \sigma t. \tag{10}$$

Dispersion relation

$$\sigma = -k/(k^2 + l^2). \tag{11}$$

Trapped baroclinic waves

$$(\tilde{u}, \tilde{v}, \tilde{h}) = (iU_m, V_m, iH_m)\, Ae^{i\theta_m} + c.c.;\ \theta_m = kx - \sigma_m t \tag{12}$$

with the dispersion relation

$$\sigma_m^3 - (k^2 + 2m + 1)\sigma_m - k = 0;\ m = 0, 1, 2, \dots\ , \tag{13}$$

m - meridional wavenumber of the trapped wave.

Equator is a wave-guide transparent for some type of waves!

For $m > 0$, and $k \leq 0$ the lower branches of (13) correspond to the equatorial Rossby waves. The upper branches describe equatorial inertia-gravity waves. $m = 0$ corresponds to Yanai (mixed Rossby-gravity) waves, $m = -1$ corresponds to dispersionless Kelvin wave. The functions $U_m, V_m = \phi_m, H_m$ are strongly localized near the equator ($y = 0$). They are expressed in terms of the parabolic cylinder functions:

$$\phi_m(y) = \frac{\mathcal{H}_m(y)e^{-\frac{y^2}{2}}}{\sqrt{2^m m! \sqrt{\pi}}}, \; U_m(y) = \frac{\sigma_m y \phi_m - k\phi'_m}{\sigma_m^2 - k^2}, \; H_m(y) = \frac{ky\phi_m - \sigma_m \phi'_m}{\sigma_m^2 - k^2}, \tag{14}$$

where $\mathcal{H}_m(y)$ are the Hermite polynomials and the prime means y-differentiation.

Baroclinic/barotropic zonal flow - exact solution of (5 - 7):

$$\bar{u} = \bar{u}(y), \;\; \bar{h} = \bar{h}(y), \;\; y\bar{u} + \bar{h}_y = 0; \;\; \bar{\psi} = \bar{\psi}(y). \tag{15}$$

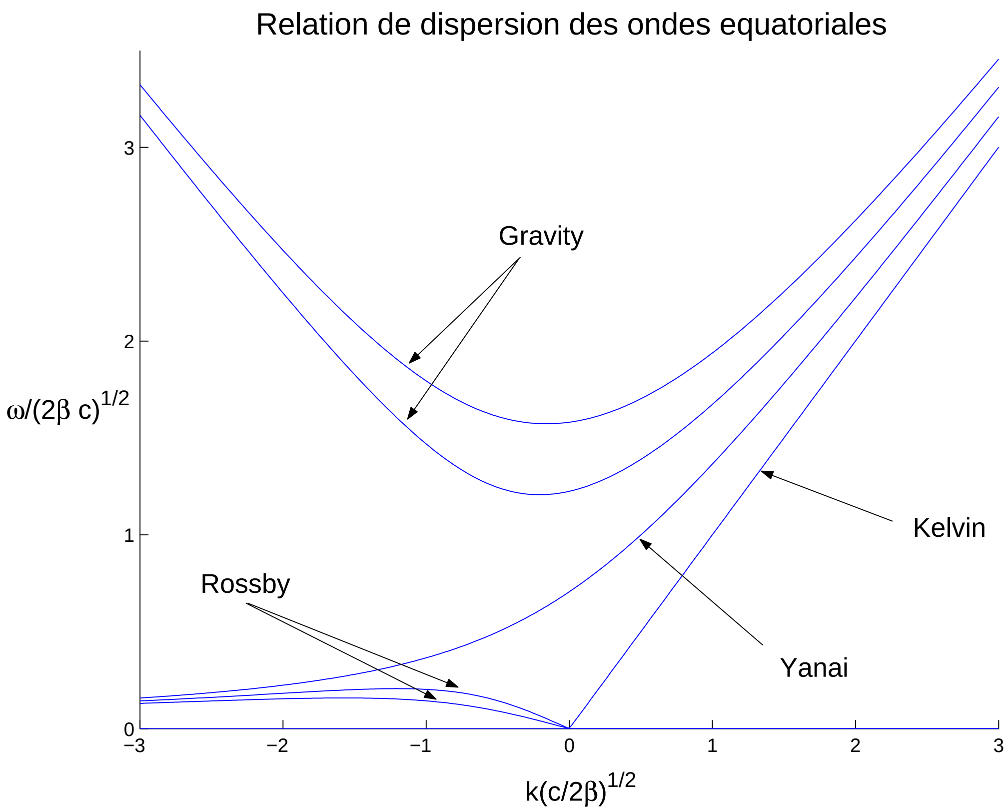

Figure 1: The dispersion curves for trapped equatorial waves. Modes with $m = -1, 0, 1, 2$ are displayed

2 The main ideas and a reminder of previous results

2.1 The philosophy of the approach

Look for weakly nonlinear interactions of the (plane) barotropic wave passing through the equator with:

1. a pair of trapped baroclinic waves

2. a trapped baroclinic wave and an equatorial zonal flow

These may produce a resonant growth of the baroclinic waves (a parametric, or sub-harmonic resonance) in the equatorial waveguide thus becoming "semi-transparent". If this is the case, study eventual nonlinear saturation and then the effects of spatial modulation.

If works, expect multiple equilibria in the resulting Landau equation (without spatial modulation) and nontrivial spatio-temporal organization in the Ginzburg-Landau (GL) type equation (with spatial modulation). A (single?) known in GFD example of trapped waves excited by free waves: beach waves (Minzoni & Whitham, 1977).

Work on parametric excitation of topographic trapped waves in progress (Reznik & Zeitlin, 2007).

2.2 Reminder of the results on three-wave resonances (Reznik & Zeitlin, 2006)

1. A pair of trapped baroclinic Rossby or Yanai waves and a barotropic wave with

$$k_1 + k_2 = k; \;\; \sigma_1 + \sigma_2 = \sigma, \tag{16}$$

   where $\sigma_{1,2}$ verify (13) for some $m_{1,2}$, and σ verifies (11) for some l do resonate. Particular case of "pure" parametric resonance: $k_1 = k_2 = k/2$. Solutions of the synchronism conditions (16) are dense in the phase-space k, l.

2. The resonant growth of baroclinic wave(s) is always saturated nonlinearly. In the $k_1 = k_2$ case Landau equation has two different stationary solutions. In the $k_1 \neq k_2$ the saturated amplitudes slowly oscillate in time.

3. A GL-type modulation equation arises in the spatially modulated $k_1 = k_2$ case

$$A_T - \frac{i}{2}\hat{\sigma}''(\hat{k})A_{XX} + LA_\psi \bar{A} + (P + iQ)\left|A\right|^2 A = 0, \tag{17}$$

giving "domain wall" and "dark soliton" patterns of spatio-temporal organisation. A pair of coupled GL-type equations arises in the spatially modulated $k_1 \neq k_2$ case.

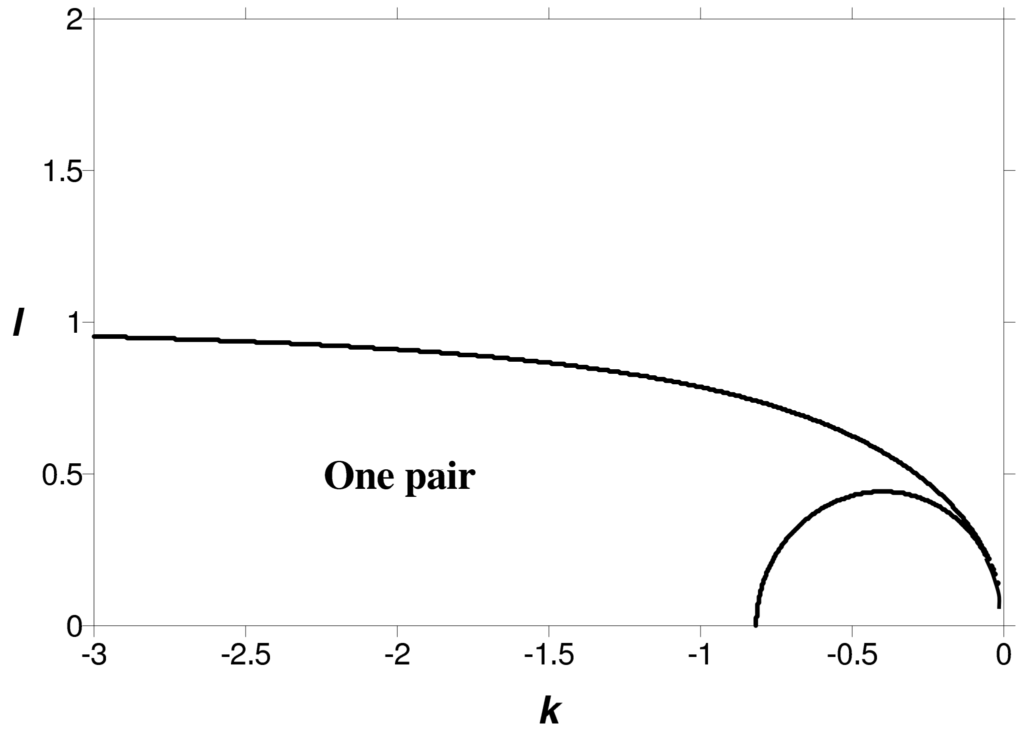

Figure 2: Typical resonance domain in the phase-space of the barotropic wave. Yanai - Rossby resonance

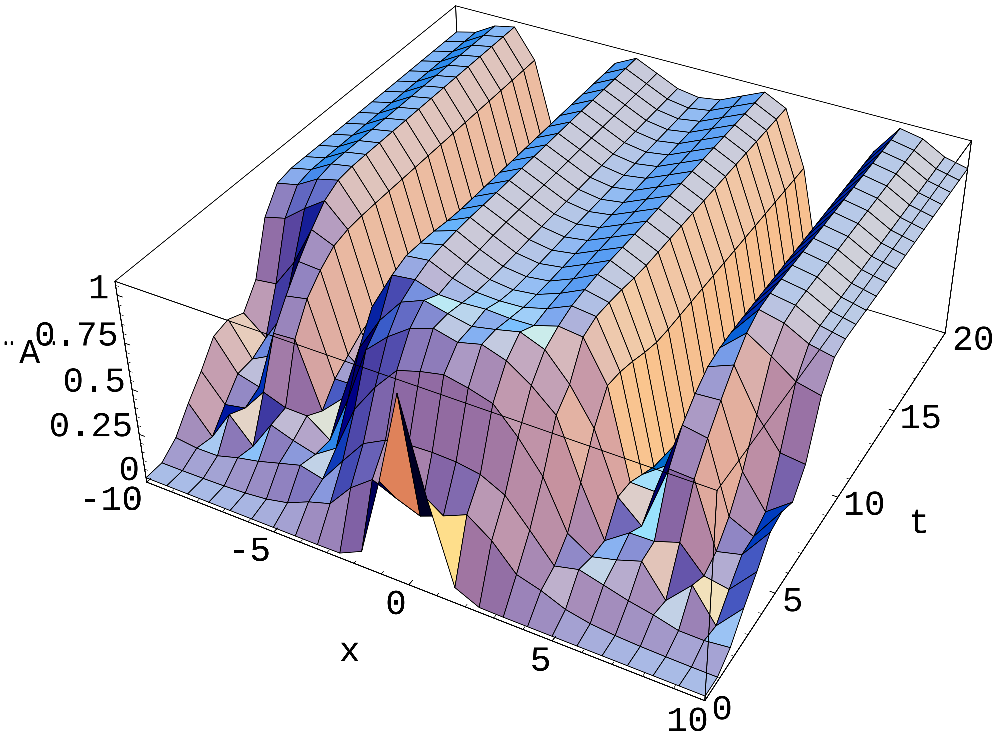

Figure 3: DNS of the GL-type equation: formation of "dark soliton" in $AbsA$

3 Barotropic-baroclinic wave resonance in the presence of a zonal flow

3.1 Synchronism conditions

The barotropic and baroclinic waves should have the same k, σ. Possible if

$$l^2 = 2m + 1 - \sigma_m^2, \tag{18}$$

For baroclinic Rossby and Yanai waves $\sigma_m < 1$ and for a given baroclinic mode m the corresponding barotropic mode exists.

- Yanai wave, $m = 0$:

$$l^2 = 1 - \sigma_0^2 = -k\sigma_0, \quad \sigma_0 = k/2 + \sqrt{1 + k^2/4}. \tag{19}$$

- Rossby wave, $m \geq 1$. In this case with high accuracy

$$\sigma_m \simeq -\frac{k}{k^2 + 2m + 1} \tag{20}$$

and

$$l^2 \simeq 2m + 1 - \frac{k^2}{(k^2 + 2m + 1)^2} \approx 2m + 1. \tag{21}$$

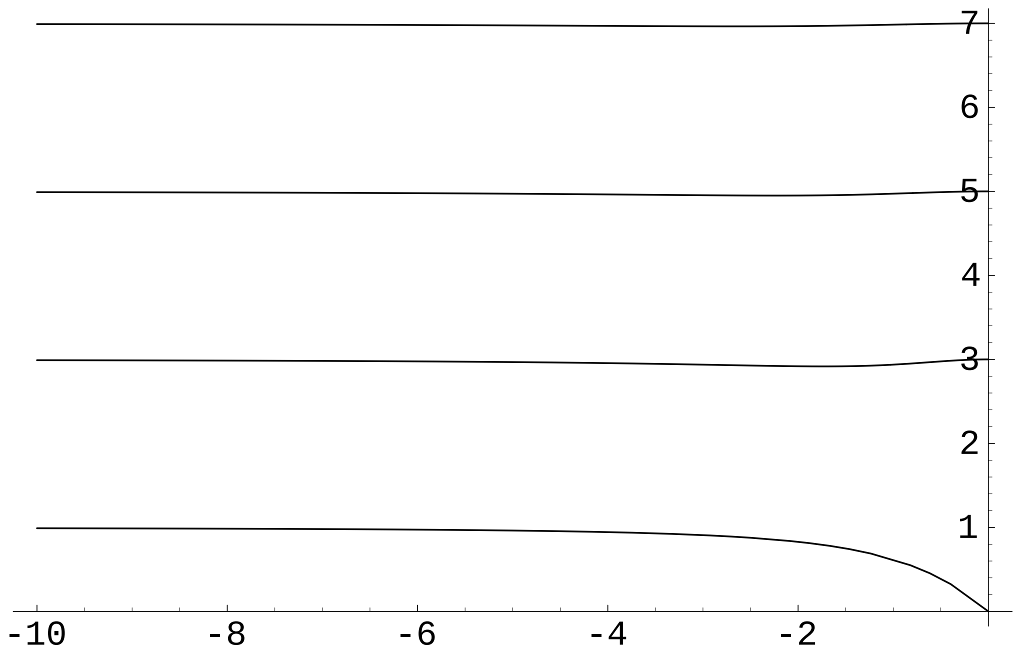

Figure 4: Solutions to synchronism conditions in the phase-space of the barotropic wave.

3.2 Removal of resonances

Forced linear system arising at each order of expansion in nonlinearity has the form:

$$\nabla^2 \psi_t + \psi_x = Q_\psi \tag{22}$$

$$u_t - yv + h_x = Q_u,\ v_t + yu + h_y = Q_v,\ h_t + u_x + v_y = Q_h. \tag{23}$$

Solution is bounded provided the following orthogonality conditions are satisfied:

$$\langle \hat{\psi} Q_\psi \rangle_{x,y,t} = 0, \tag{24}$$

$$\int_{-\infty}^{\infty} dy\, \langle \hat{u} Q_u + \hat{v} Q_v + \hat{h} Q_h \rangle_{x,t} = 0, \tag{25}$$

where $\hat{\psi}, \hat{u}, \hat{v}, \hat{h}$ is an arbitrary bounded solution of the homogeneous equations (22), (23) and the angles denote the averaging:

$$\langle ... \rangle_x = \lim_{L_x \to \infty} \frac{1}{2L_x} \int_{-L_x}^{L_x} dx \, ... \tag{26}$$

In our problem, the source terms have the form:

$$Q_{\psi,...,h} = \sum_q Q^q_{\psi,...,h}(y) e^{i(k_q x - \sigma_q t)}, \tag{27}$$

with $Q^q_{\psi,...,h}(y)$ rapidly decaying at $y \to \pm\infty$. For such $Q_{\psi,...,h}$ the conditions (24), (25) are not only necessary, but also sufficient for existence of bounded solutions to (22), (23) if $k_q \neq 0$, $\sigma_q \neq 0$.

3.3 Results of straightforward perturbation expansion

Multi-timescale expansion:

$(\psi, u, v, h) = (\psi^{(0)}, u^{(0)}, v^{(0)}, h^{(0)})(x, y, t, T, ...) + \epsilon(\psi^{(1)}, u^{(1)}, v^{(1)}, h^{(1)})(x, y, t, T, ...)$

$(\psi^{(0)}, u^{(0)}, v^{(0)}, h^{(0)}) = (\tilde{\psi}^{(0)}, \tilde{u}^{(0)} + \bar{u}^{(0)}, \tilde{v}^{(0)} + \bar{v}^{(0)}, \tilde{h}^{(0)} + \bar{h}^{(0)})$, Tilde - waves, bar - zonal flow. Lowest order results:

- Barotropic wave remains unchanged: A_ψ does not depend on T,
- Zonal flow $\bar{u}^{(0)}$, $\bar{h}^{(0)}$ does not change in time (Eliassen-Palm),
- Slow-time evolution of the amplitude of the baroclinic wave:

$$a_0 A_T + iqLA = -kL_\psi A_\psi. \quad (28)$$

Solution:

$$A = \frac{ikL_\psi}{qL} A_\psi + C_0 e^{-\frac{qL}{a_0}T}, \; C_0 = const. \tag{29}$$

The coefficients:

$$
\begin{aligned}
a_0 &= \int_{-\infty}^{\infty} dy\,(U_m^2+\phi_m^2+H_m^2),\\
L_\psi &= \int_{-\infty}^{\infty} dy\,e^{ily}\left[\bar{h}_y^{(0)}H_m-(k\phi_m+2ilU_m+U_{m_y})\bar{u}^{(0)}\right],\\
L &= \int_{-\infty}^{\infty} dy\,\Big(\left[k(U_m^2+\phi_m^2+H_m^2)+(U_m\phi_m)_y\right]\bar{u}^{(0)}\\
&+ (kH_mU_m+\phi_mH_{m_y})\bar{h}^{(0)}\Big),
\end{aligned}
\tag{30}
$$

If $q\,L = 0$ then linear growth:

$$A = \frac{-kL_\psi}{a_0} A_\psi T + C_0 \qquad (31)$$

For resonant growth it is necessary that:

- either $q = 0$ - layers of equal depth,
- or $L = 0$ - special zonal flow

Nevertheless growth always results for slightly *detuned* frequencies: $\sigma_{bt} = \sigma_{bc} + \epsilon\delta$, with $\delta = \frac{qL}{a_0}$. In what follows $q = 0$ for technical simplicity.

4 Nonlinear saturation of growth in the case of barotropic-baroclinic-meanflow resonance.

4.1 General modulation equation

Method of studying saturation: rearrangement of asymptotic expansions. Solution is sought in the form:

$$\begin{aligned} \psi^{(0)} &= A_\psi e^{i(\theta+ly)} + c.c. + \epsilon^\gamma \bar{\psi}(y) + \psi^{(1)}(x,y,t,T_{\beta'},\epsilon), \\ (u^{(0)},v^{(0)},h^{(0)}) &= \epsilon^\alpha(\bar{u}^{(0)},0,\bar{h}^{(0)})(y,T_{\alpha'}) + \epsilon^\beta(iU_m,\phi_m,iH_m)A(T_{\beta'})e^{i\theta} \\ &+ (u^{(1)},v^{(1)},h^{(1)})(x,y,t,T_{\alpha'},T_{\beta'},\epsilon) + c.c, \end{aligned} \tag{32}$$

where $T_{\alpha',\beta'} = \epsilon^{\alpha',\beta'} t$, $\alpha',\beta' > 0$, and it is supposed that the mean flow is sufficiently intense: $1 < \alpha \leq 0$, $\gamma \leq 0$.

Parameters α, γ are fixed, and the value of β which determines the saturated baroclinic amplitude ($\beta < 0$ in such case) is to be found. The correction to the barotropic wave is determined from:

$$\nabla^2 \psi_t^{(1)} + \psi_x^{(1)} = \epsilon s \left[-(\partial_{xx} - \partial_{yy})(u^{(0)} v^{(0)}) + \partial_{xy}(u^{(0)2} - v^{(0)2}) \right], \tag{33}$$

It is this correction which will give either linear or nonlinear *saturation* of the trapped wave growth via, resp., interaction with zonal flow and the trapped wave, or triad interaction with the trapped wave .

Eliminating resonances while finding the baroclinic correction $(u^{(1)}, v^{(1)}, h^{(1)})$ leads to the modulation equation for A:

$$\epsilon^{\beta'} a_0 A_{T_{\beta'}} + \epsilon^{2+2\alpha}(p+iq)A + \epsilon^{1+\gamma} i L_{\psi_0} A + \epsilon^{2+2\beta}(P+iQ)|A|^2 A = -\epsilon^{1+\alpha-\beta} k L_{\psi}. \tag{34}$$

One recognizes in the r.h.s. the previously studied resonant forcing of the baroclinic wave.

- p, q arise from wave-mean-mean interactions,
- L_{ψ_0} arises from wave -barotropic flow interaction,
- P, Q arise from three-wave interaction.

4.2 Analysis of possible regimes and canonical modulation equation

Analysis of the modulation equation in the *absence of the barotropic component of the zonal flow* shows that at $\alpha \leq -\frac{1}{2}$ the linear saturation dominates, while at $\alpha \geq -\frac{1}{2}$ it is the nonlinear one. In both cases $\beta \geq -\frac{1}{2}$, and thus the limiting value of A do not exceed $\epsilon^{-\frac{1}{2}}$, which is achieved at $\alpha = \beta = -\frac{1}{2}$. The slow time-scale is determined by $\beta' = 1 + \alpha - \beta$.

In the *presence of the barotropic component of the zonal flow*, if this latter is weak with respect to the baroclinic flow, its role is reduced to changing the linear saturation coefficients. If the barotropic flow is strong it acts similar to non-zero q in the previous analysis and, thus a detuning is necessary to have growing and then saturating solutions.

"Optimal" case $\alpha = \beta = -\frac{1}{2}$, no barotropic flow (detuning otherwise). Modulation equation:

$$A_{T_2} + (\bar{p} + i\bar{q})A + (\bar{P} + i\bar{Q})|A|^2 A = -\frac{kL_\psi}{a_0} A_\psi, \quad \bar{P} \geq 0, \bar{p} > 0. \tag{35}$$

The real parts of $\bar{p}, \bar{P}$ are:

$$\bar{p} = \frac{1}{8|l|\sigma} \left| \int_{-\infty}^{+\infty} dy\, F_1(y) e^{ily} \right|, \tag{36}$$

$$\begin{aligned} \bar{P} &= \frac{1}{16|\bar{l}|\sigma} \left| \int_{-\infty}^{+\infty} dy\, F_2(y) e^{ily} \right|, \text{ if } \bar{l}^2 = l^2 - 3k^2 < 0, \\ \bar{P} &= 0, \text{ if } \bar{l}^2 = l^2 - 3k^2 > 0. \end{aligned} \tag{37}$$

Here

$$F_1 = (\phi \bar{u}_0)'' - 2k(U\bar{u}_0)' + k^2 \phi \bar{u}_0, \quad F_1 = (\phi U)'' - 2k(U^2 + \phi^2)' + 4k^2 \phi U, \tag{38}$$

and the index m is omitted in meridional wave structure functions (14). We do not give the expression for $\bar{q}$, $\bar{Q}$ which have similar structure but are rather cumbersome.

Most important:

$$\bar{p} \geq 0, \ \bar{P} \geq 0, \tag{39}$$

and, hence, the corresponding terms produce saturation of A Saturation due to p will be called "linear" and that due to P "nonlinear".

4.3 Analysis of saturated solutions

By renormalizing A and T the number of relevant parameters in (35) may be reduced:

$$A_T + e^{i\xi} A + e^{i\eta} |A|^2 A = c_0 \, |A_\psi| \equiv c, \;\; Im \, c_0 = 0. \tag{40}$$

Looking for time-independent solutions, a cubic equation for the square modulus of A readily follows:

$$|A|^6 + 2\cos\chi |A|^4 + |A|^2 - c^2 = 0, \;\; \chi = \xi - \eta, \tag{41}$$

which has either three positive roots, or a single positive root. An elementary analysis shows that necessary and sufficient conditions of the existence of three roots are:

$$\cos\chi < -\frac{\sqrt{3}}{2}, \;\; F(x_+) < c^2 < F(x_-), \tag{42}$$

where

$$F(x) = x^3 + 2\cos\chi + x, \quad x_\pm = -\frac{2}{3} \pm \sqrt{\frac{4}{9}\cos^2\chi - \frac{1}{3}}. \qquad (43)$$

Analysis of stability of a stationary solution shows that:

- In case of a single root, it is always stable
- In the case of three roots, the largest and the smallest are stable, while the intermediate one is unstable.

Stable solutions are attracting in the phase space of ReA, ImA.

Remark: depending on the coefficients, zero may lie in the domain of attraction of either smaller or larger root.

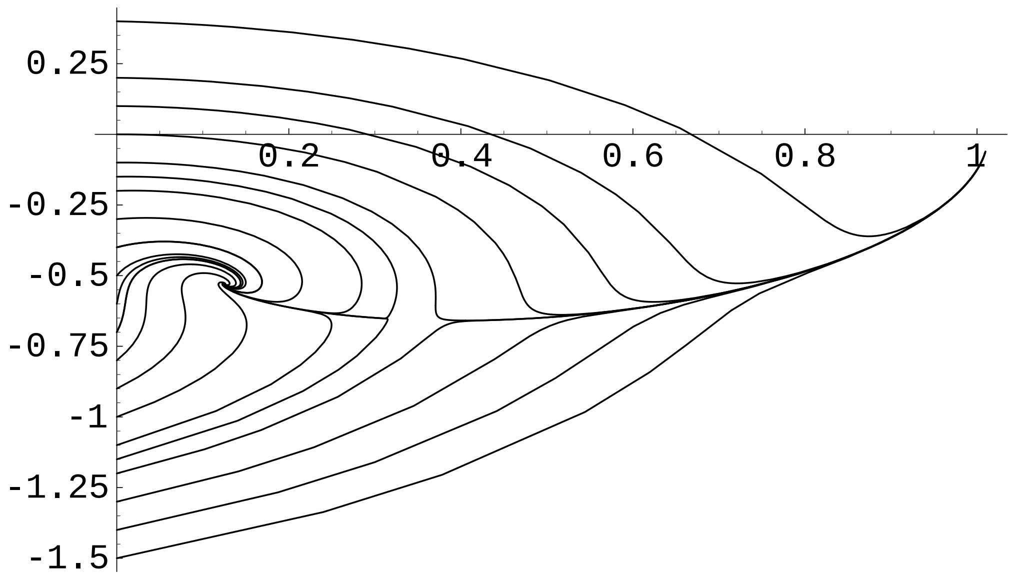

Figure 5: The phase portrait of the system (40) with $\eta = -.4\pi$, $\xi = 19\pi/20$, $c = .4$

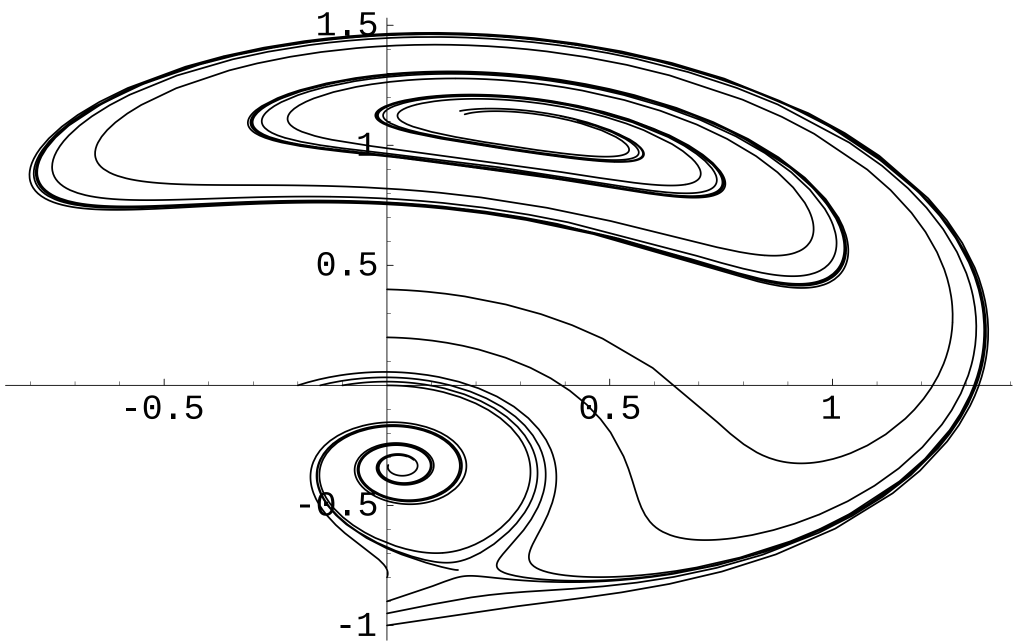

Figure 6: The phase portrait of the system (40) with $\eta = \pi/2$, $\xi = 19\pi/20$, $c = .3$

5 The effects of spatial modulation

We consider a (typical) case of strong baroclinic zonal current $\sim \epsilon^{-\frac{1}{2}}$ in the absence of the barotropic zonal current $\bar{\psi} \equiv 0$, and introduce slow spatial modulation in the zonal direction of the baroclinic *and* barotropic waves with the scale $X = \epsilon^{\frac{1}{2}} x$. ($\alpha = \beta = -\frac{1}{2}$) The technicalities of the analysis follow Reznik and Zeitlin (2006). The "synthetic" modulation equations for A and A_ψ follow:

$$\left(\partial_{T_1} + c_g^{bt} \partial_X\right) A_\psi - \epsilon^{\frac{1}{2}} \frac{i}{2} \left(\sigma^{bt}\right)'' \partial^2_{XX} A_\psi = 0, \qquad (44)$$

$$\begin{aligned} \left(\partial_{T_1} + c_g^{bc} \partial_X\right) A \quad &+ \quad \epsilon^{\frac{1}{2}} \left[-\frac{i}{2} \left(\sigma^{bt}\right)'' \partial^2_{XX} A + (\bar{p} + i\bar{q}) A \right. \\ &+ \quad \left. (P + iQ)|A|^2 A\right] = -\epsilon^{\frac{1}{2}} c_0 A_\psi. \end{aligned} \qquad (45)$$

Here $T_1 = \epsilon^{\frac{1}{2}} t$, $\sigma^{bt,bc}$ are frequencies of the barotropic and the baroclinic waves, as expressed via their corresponding dispersion relations, $c_g^{bt,bc} = \left(\sigma^{bt,bc}\right)'$ are the corresponding zonal group velocities, and prime denotes differentiation with respect to zonal wavenumber k.

The group velocity of the Yanai wave may differ significantly from the group velocity of the barotropic Rossby wave of the same frequency. E. g. for zonally long waves, $k \ll 1$, $c_g^{bc} \approx \frac{1}{2} \ll c_g^{bt} \approx -\frac{1}{k}$. On the contrary, the group velocities of the baroclinic and the barotropic Rossby waves of the same frequency are practically the same.

In the former case, the only situation where barotropic and baroclinic waves have possibility to interact is that of "gentle" modulation when the fields depend on $X_1 = \epsilon x$, and not on X, and on $T_2 = \epsilon t$, and not on T_1. In this case dispersion effects are weak, and

$$\partial_{T_2} A_\psi + c_g^{bt} \partial_{X_1} A_\psi = 0, \quad (46)$$

$$\partial_{T_2} A + c_g^{bc} \partial_{X_1} A + (\bar{p} + i\bar{q})A + (\bar{P} + i\bar{Q})|A|^2 A = -c_0 A_\psi. \quad (47)$$

In the latter case by choosing the reference frame moving with the common group velocity we get:

$$\partial_{T_1} A_\psi - \frac{i}{2} \left(\sigma^{bt}\right)'' \partial^2_{XX} A_\psi = 0, \tag{48}$$

$$\partial_{T_1} A - \frac{i}{2} \left(\sigma^{bt}\right)'' \partial^2_{XX} A + (\bar{p} + i\bar{q})A + (\bar{P} + i\bar{Q})|A|^2 A = -c_0 A_\psi. \tag{49}$$

This is a GL-equation for A forced by the wave-packet of barotropic waves which, in turn, is subject to dispersion.

Finally, if there is no spatial modulation of the barotropic wave (plane barotropic wave occupying the whole equatorial plane) we get, by changing the reference frame, the equation (49) with constant A_ψ.

Remark1: For $P = 0$, which is the case of short enough waves

$$|k| > \frac{1}{2\sqrt{3}}, \quad |k| \geq \sqrt{\frac{2m+1}{3}}, m = 1, 2, ... \tag{50}$$

for Yanai and Rossby waves, respectively, by rescaling A with time-depending phase a nonlinear Schrodinger equation with oscillating forcing and linear damping results.

Remark2: In the case with two different X - independent stationary solutions the domain-wall like structures (spatio-temporal organization) are expected.

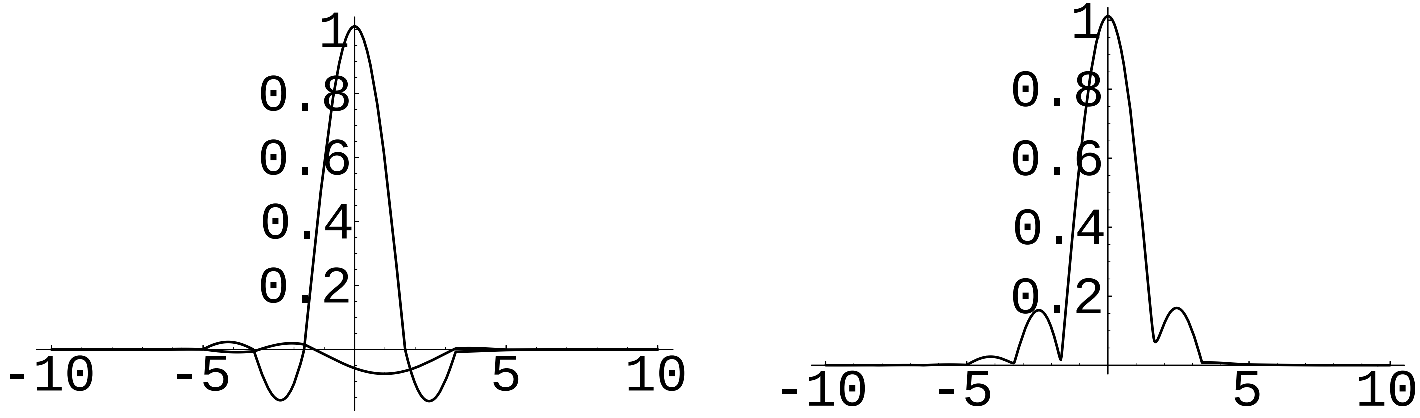

Figure 7: "No-dispersion" case, DNS of (46), (47): profiles of ReA, ImA (left panel), and $AbsA$ (right panel) at $T_2 = 30$ in a reference frame moving with the barotropic wave; $\eta = -.4\pi$, $\xi = 19\pi/20$, the barotropic wave is Gaussian with max. amplitude .4 covering the domain of attraction of both stationary states.

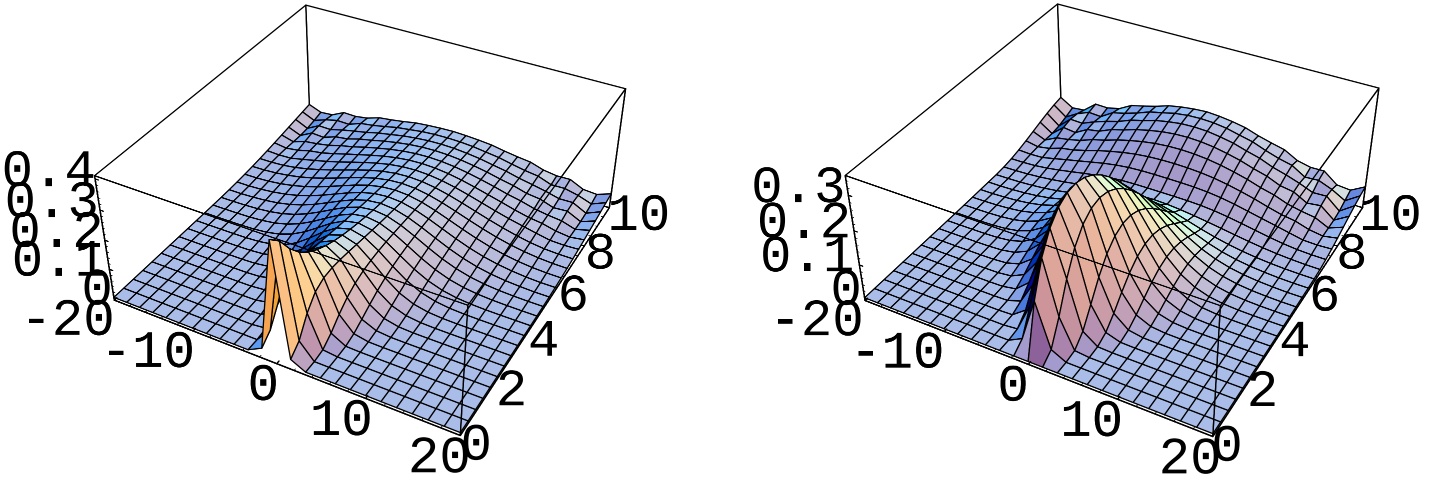

Figure 8: "Dispersion" case, DNS of (48), (49) : spatio-temporal evolution of A_ψ (left panel), and A (right panel); $\eta = -.4\pi$, $\xi = 19\pi/20$

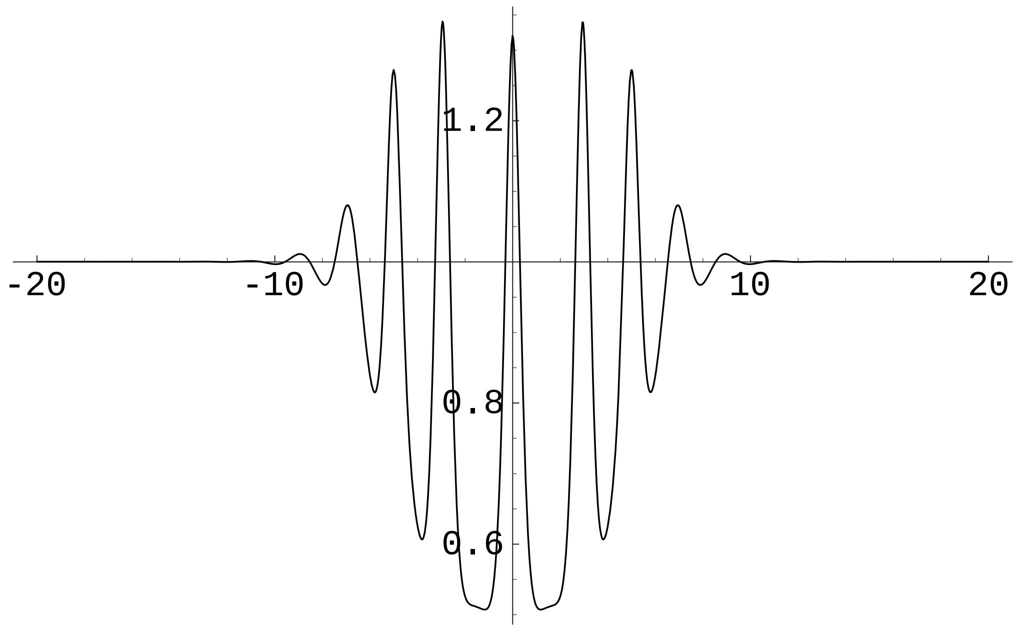

Figure 9: Plane barotropic wave case, DNS of (49) with constant A_ψ such that two different stationary solutions exist: section of $AbsA$ at $T = 30$; $\eta = -.4\pi$, $\xi = 19\pi/20$

6 Conclusions

- Baroclinic zonal current at the equator acts as a resonator: it responds to certain incoming barotropic waves by amplifying (from the pre-existing noise) the trapped baroclinic Yanai and/or Rossby waves which grow to significant amplitudes, and then are nonlinearly saturated.
- In the certain range of parameters, multiple equilibria of the modulation equation exist, leading to bifurcations in the initial values of the baroclinic amplitudes and to spatio-temporal organization.