KOLMOGOROV WEAKLY TURBULENT SPECTRA OF SOME TYPES OF DRIFT WAVES IN PLASMAS

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Received 22 April 1988; revised manuscript received 11 August 1988; accepted for publication 16 September 1988
Communicated by R.C. Davidson

A possibility of a realization of Kolmogorov (power-law) spectra of weak turbulence of lower-hybrid drift waves in plasmas with hot ions and for some types of electron gradient waves in inhomogeneous magnetized plasmas is shown. The turbulence due to a vector or scalar nonlinearity is discussed.

1. Introduction

The analytical approach to the problem of Kolmogorov spectra of weak drift-wave turbulence in an inhomogeneous magnetized plasma has been developed in ref. [1]. Two examples have been discussed there of waves with which may be connected such a turbulence: waves governed by the Hasegawa–Mima equation [2] and three-dimensional short-wavelength drift-Alfvén waves governed by a set of nonlinear equations obtained in ref. [3]. It has been noted in ref. [4] that two-dimensional Kolmogorov turbulence may be realized also on the branch of long-wavelength (compared to the ion Larmor radius) ion drift waves propagating transversely to a magnetic field.

Usually the Hasegawa–Mima equation is associated with oblique ($k_z \neq 0$, k_z is the longitudinal wavenumber) electrostatic electron drift waves (see ref. [2]). In the present paper we pay attention to the fact that such a type of nonlinear equations may be also used for a description of some other drift-wave types: lower-hybrid drift (LHD) waves in a plasma with hot ions (section 2), electrostatic electron gradient waves (section 3) and electron electromagnetic (compressible) flute waves (section 4). It follows that a turbulence with Kolmogorov spectra as found in ref. [1] may be realized in the case of the above mentioned waves. Besides that, we discuss three-dimensional electron electromagnetic (compressible) gradient waves (section 5) and show that these waves are governed by nonlinear equations of the same structure as in the case of short-wavelength drift-Alfvén waves. Consequently, a three-dimensional turbulence with energy spectra as found in ref. [1] may be related with these waves.

In refs. [1,2] as in sections 2–5 we discuss wave interactions due to a vector nonlinearity (see the terminology in ref. [5]). The Kolmogorov drift-wave turbulence can be realized also in problems with a scalar nonlinearity. An example of such a turbulence is discussed in section 6.

2. LHD waves in a plasma with hot ions

We use the term "LHD waves in a plasma with hot ions" for the designation of waves with dispersion

$$\omega_k = \frac{k_y V_{ni}}{1+k_\perp^2 \rho_{0e}^2}, \qquad (1)$$

where ω_k is the wave frequency with wave vector $\boldsymbol{k}$, k_y the wavenumber along the drift direction y, $k_\perp=(k_x^2+k_y^2)^{1/2}$ the transverse wavenumber, k_x the wavenumber along the plasma inhomogeneity direction x, $V_{ni}=cT_i\kappa_n/eB_0$ the ion drift velocity due to the density gradient, $\kappa_n=\partial \ln n_0/\partial x$, n_0 the plasma number density, T_i the ion temperature, $\boldsymbol{B}_0 \| z$ the equilibrium magnetic field, $\rho_{0e}^2=T_i/m_e\omega_{Be}^2$ the square of the electron Larmor radius calculated at the ion temperature, m_e the electron mass, e the ion charge, c the velocity of light. For the designation of waves of type (1) the term "short-wavelength ion-drift waves in a plasma with cold electrons" is also used (see, e.g., ref. [3] and references therein).

Waves of type (1) are of short wavelength compared to ions, i.e. $k_\perp\rho_i \gg 1$, where ρ_i is the ion Larmor radius. Therefore, the total ion density in the case discussed has the Boltzmann form (see for details ref. [6])

$$n=n_0 \exp(-e\phi/T_i), \qquad (2)$$

where ϕ is the electrostatic potential. Taking into account that in these waves $k_z\to 0$ and substituting (2) into the electron continuity equation we arrive at the equation for ϕ:

$$\frac{\partial}{\partial t}(\phi-\rho_{0e}^2\Delta_\perp\phi)+V_{ni}\frac{\partial\phi}{\partial y}$$
$$-\rho_{0e}^2\frac{c}{B_0}[\nabla\phi\times\nabla]_z\Delta_\perp\phi=0, \qquad (3)$$

where $\Delta_\perp=\partial^2/\partial x^2+\partial^2/\partial y^2$. Eq. (3) coincides to within replacements $V_{ni}\to V_{ne}=-cT_e\kappa_n/eB_0$ (V_{ne} is the electron drift velocity due to the density gradient, T_e the electron temperature), $\rho_{0e}^2\to\rho_{0i}^2=T_e/m_i\omega_{Bi}^2$ (ρ_{0i} is the ion Larmor radius calculated at the electron temperature, m_i the ion mass, ω_{Bi} the ion cyclotron frequency) with the Hasegawa–Mima equation for oblique drift waves [2]. Consequently, mathematically the problem of the turbulence of LHD waves in a plasma with hot ions is reduced to the problem discussed in ref. [1]. According to ref. [1] energy spectra of these waves with $k_x \gg k_y$ and in the case $k_x\rho_{0e}\gg 1$ have the form

$$W_k \propto (k_y^{-3/2}k_x^{-2},\, k_y^{-3/2}k_x^{-3}), \qquad (4)$$

or, in the case when $k_x\rho_{0e}\ll 1$,

$$W_k \propto (k_y^{-3/2}k_x^{-4},\, k_y^{-3/2}k_x^{-3}). \qquad (5)$$

3. Electron electrostatic gradient waves

According to ref. [7], the branch of electrostatic waves with $k_z=0$ exists in a strongly inhomogeneous low pressure plasma with the dispersion law

$$\omega_k=\frac{k_y\kappa_n}{k_\perp^2}\frac{\omega_{pe}^2}{\omega_{Be}(1+\omega_{pe}^2/\omega_{Be}^2)}, \qquad (6)$$

where ω_{pe} is the electron plasma frequency. We note that if $T_i\to\infty$ in (1) and $\omega_{pe}\gg\omega_{Be}$ in (6), then in both cases we have $\omega_k=-k_y\kappa_n\omega_{Be}/k_\perp^2$. Therefore, it is clear that waves of types (1) and (6) may be considered as the limiting cases of the same branch of oscillations. A nonlinear equation for waves of type (6) can be obtained by using the Poisson equation and the electron continuity equation. As a result we have

$$\left(\frac{\partial}{\partial t}+\frac{c}{B_0}[\nabla\phi\times\nabla]_z\right)\Delta_\perp\phi$$
$$-\frac{\kappa_n\omega_{pe}^2}{\omega_{Be}(1+\omega_{pe}^2/\omega_{Be}^2)}\frac{\partial\phi}{\partial y}=0. \qquad (7)$$

One can see that (7) has the same structure as (3) when $\rho_{0e}^2k_\perp^2\gg 1$. Consequently, eq. (5) is a variant of the Hasegawa–Mima equation [2] and, hence the Kolmogorov turbulence of these waves with $k_x\gg k_y$ is described by (4).

4. Electron electromagnetic (compressible) flute waves

We call electron electromagnetic (compressible) flute waves, those propagating transversely to the magnetic field ($k_z=0$) in a strongly inhomogeneous magnetized plasma with dispersion [9]

$$\omega_k=\frac{k_y u_{ne}}{1+c^2k_\perp^2/\omega_{pe}^2}, \qquad (8)$$

where $u_{ne}=c\kappa_nB_0/4\pi e n_0$. According to ref. [10], a nonlinear equation for waves of type (8) has the form

$$\left(\frac{\partial}{\partial t}+u_{ne}\frac{\partial}{\partial y}\right)\phi$$
$$-\frac{c^2}{\omega_{pe}^2}\left(\frac{\partial}{\partial t}+\frac{c}{B_0}[\nabla\phi\times\nabla]_z\right)\Delta_\perp\phi=0. \qquad (9)$$

One can see that eq. (9) has the same structure as (3). The Kolmogorov turbulence connected with waves of type (8) is characterized by (4) or (5).

5. Oblique electron electromagnetic (compressible) gradient waves

According to ref. [10], in a strongly inhomogeneous plasma we have the branch of purely electronic oscillations with $k_z\neq 0$ and with the frequency

$$\omega_k=k_y u_{ne}+\frac{c^4\omega_{Be}}{\omega_{pe}^4 u_{ne}}\frac{k_z^2 k_\perp^2}{k_y}. \qquad (10)$$

The second term on the right hand side of (10) is a correctional one, so (10) is true at sufficiently small $ck_\perp/\omega_{pe}$. In (10) we neglected the transverse dispersion term compared to the longitudinal one. In the opposite case we have instead of (10) the long-wavelength limit of eq. (8). For the investigation of nonlinear waves of type (8) we proceed from the equations of electron hydrodynamics obtained in ref. [10]. So we have the set of nonlinear equations

$$\left(\frac{\partial}{\partial t}+u_{ne}\frac{\partial}{\partial y}\right)\phi$$
$$-\frac{c^3\omega_{Be}^2}{\omega_{pe}^4}\left(\frac{\partial}{\partial t}-\frac{1}{B_0}[\nabla A\times\nabla]_z\right)\Delta_\perp A=0, \qquad (11)$$

$$\frac{\partial A}{\partial t}+c\left(\frac{\partial}{\partial t}-\frac{1}{B_0}[\nabla A\times\nabla]_z\right)\phi=0, \qquad (12)$$

where A is the longitudinal component of the vector potential. These equations have the same structure as the equations for short-wavelength drift-Alfvén waves obtained in ref. [3] when $T_i\to\infty$. Taking the above and the results of ref. [1] into account, we conclude that the Kolmogorov turbulence characterized by formula (30) of ref. [1] is connected with waves of type (10).

6. Kolmogorov drift-wave turbulence due to a scalar nonlinearity

At sufficiently small k the nonlinear equation of type (3) should be replaced, generally speaking, by

$$\left(\frac{\partial}{\partial t}+V_*\frac{\partial}{\partial y}\right)\phi-\rho_0^2\frac{\partial^3\phi}{\partial t\partial x^2}+C\frac{\partial\phi^2}{\partial y}=0. \qquad (13)$$

Here C is a constant, V_* a characteristic velocity for the problem (in the cases discussed above V_* equals V_{ni}, V_{ne} or u_{ne}), ρ a characteristic space scale (ρ_{0e}, ρ_{0i} or c/ω_{pe}). In (13) we assumed that $\partial/\partial x\gg\partial/\partial y$. The term with ϕ^2 in (13) corresponds to the scalar nonlinearity. In cases of oblique drift waves or LHD ones the constant C is proportional to the temperature gradient (see, e.g., refs. [5,10]), but in the case of waves of type (8) this constant can be obtained by using the results of ref. [10]. The characteristic value $k_\perp\simeq|k_x|$ at which the transition from (3) to (13) is performed can be obtained by a comparison of the nonlinear terms of these equations.

Starting from (13) and using the approach of ref. [1], we find that the stationary Kolmogorov turbulence is characterized by energy spectra

$$W_k\propto(k_y^{-3/2}k_x^{-1},\,k_y^{-3/2}), \qquad (14)$$

where $W_k\propto|\phi_k|^2$. When k_x increases spectra (14) go over into spectra (5).

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