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ON THE NONLOCAL INTERACTION WITH ZONAL FLOWS IN TURBULENCE OF DRIFT AND ROSSBY WAVES

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Theory of nonlocal weak turbulence of drift and Rossby waves [1,2] is extended to make account of the process of spectrum redistribution over scales owing to nonlocal interaction with zonal flows. Correspondent generalization of the equation for turbulent spectrum evolution is obtained. According to this equation the nonlocal interaction with zonal flows leads to a diffusion along mussel-like curves. These curves appear to coincide with the contours of spectral density of the new invariant found in Ref.[5].

The notion about nonlocal interaction in turbulence of drift and Rossby waves was introduced in Refs.[1,2]. Considering the evolution of drift turbulence spectrum to be determined by the interactions with large scales and with the turbulence of zonal flows and not by near scales' interactions (see the basis for such a supposition in Refs.[1,3]), and considering the turbulence to be weak, the following equation for spectrum evolution was derived in Ref.[2]:

$$\frac{\partial n_{\vec{k}}}{\partial t} = I^0_{ls} + I^0_{zf}, \tag{1}$$

where

$$I^0_{ls} = \frac{\partial \Omega}{\partial k_x}\left(\frac{\partial}{\partial k_y}\left(S\frac{\partial n_{\vec{k}}}{\partial k_y}\right)_\Omega\right)_\Omega, \tag{2}$$

$$I^0_{zf} = Y[n(k_x,-k_y) - n(k_x,k_y)], \tag{3}$$

$$S = \left(\frac{\partial \Omega}{\partial k_x}\right)^{-2}\int_{p\ll 1,k} 4\pi\left[\left|V_{\vec{k},\vec{p},-\vec{k}-\vec{p}}\right|^2 n_{\vec{p}}\right]_{p_x=-\alpha p_y} p_y^2 dp_y, \tag{4}$$

$$\alpha = \frac{\partial \Omega}{\partial k_y}\Big/\frac{\partial \Omega}{\partial k_x},$$

$$Y = \left|\frac{\partial \Omega}{\partial k_y}\right|^{-1}\int_{|q_x|\ll k_x} 8\pi\left[\left|V_{\vec{k},\vec{q},-\vec{k}-\vec{q}}\right|^2 n_{\vec{q}}\right]_{q=(q_x,-2k_y)} dq_x,$$

$$\Omega = \Omega(\vec{k}) = k_x - \omega_{\vec{k}}.$$

Here $n_{\vec{k}}$ is the spectrum of wave action, $\vec{k} = (k_x, k_y)$ is a wave vector, $\omega_{\vec{k}} = k_x/(1+k^2)$ is the dispersion low of drift and Rossby waves, operator $(\partial/\partial k_y.)_\Omega$

means k_y-derivative under the constant value of function $\Omega = \Omega(\vec{k})$;$V_{\vec{k}_1,\vec{k}_2,\vec{k}_3}$ is a matrix element, which can be different depending on particular physical situation [2].

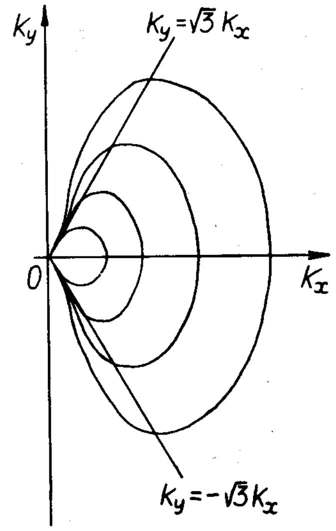

Fig.1

The first term in Eq.(1), I^0_{ls}, corresponds to the main order in the expansion of nonlocal interaction with large scales over small values of wave vectors $\vec{p}$ of these scales; while the second one, I^0_{zf}, is the main contribution of interactions with the turbulence of zonal flows (when expanding it over the small parameter q_x/k_x, where q_x, k_x are the wave vectors of zonal flow's turbulence and the turbulence under consideration, correspondingly).

According to Eqs.(1),(2) interaction with large scales leads to the spectrum redistribution proceeding independently on each of the curves $\Omega(\vec{k}) = const.$ The nonlocal interaction with the turbulence of zonal flows in the order retained in Eqs.(1),(3) doesn't contribute to a spectrum redistribution among the scales possessing different values $|k_y|$; it influences only the evolution of asymmetric (relatively k_x-axis) spectra giving rise to the symmetrization of these spectra. In other words, the spectrum redistribution on $|k_y|$ due to interaction with zonal flows is a more slow process, than its symmetrization relatively k_x-axis; this redistribution is described by the next orders in the expansion over q_x/k_x. Jet we shall show below, that the rate of such redistribution can be greater than the one caused by interaction with large scales.

Starting with the kinetic equation for waves [4]:

$$\frac{\partial n_{\vec{k}}}{\partial t} \doteq St[n],$$

$$St[n] = \int_{k_{1x},k_{2x}>0} (R_{012} - R_{102} - R_{210}) dk_1 dk_2,$$

where

$$R_{012} = 2\pi |V_{\vec{k},\vec{k_1},\vec{k_2}}|^2 \delta(\vec{k} - \vec{k_1} - \vec{k_2}) \delta(\omega_{\vec{k}} - \omega_{\vec{k_1}} - \omega_{\vec{k_2}}) [n_{\vec{k_1}} n_{\vec{k_2}} - n_{\vec{k}} n_{\vec{k_1}} - n_{\vec{k}} n_{\vec{k_2}}],$$

and retaining the two main terms in the expansion over q_x/k_x (where $q_x = k_{1x}$ or $q_x = k_{2x}$ depending on what vector, $\vec{k_1}$ or $\vec{k_2}$, lies in zonal flows' region), one can obtain more precize (than in Eq.(1)) expression for the interaction with zonal flows:

$$I_{zf} = I^0_{zf} + I^1_{zf}, \tag{5}$$

where

$$I^1_{zf} = \frac{\partial \phi}{\partial k_x} \left(\frac{\partial}{\partial k_y} \left(Z \frac{\partial n_{\vec{k}}}{\partial k_y} \right)_{\phi} \right)_{\phi}, \tag{6}$$

$$Z = \left| \frac{\partial \Omega}{\partial k_y} \right|^{-1} \left(\frac{\partial \phi}{\partial k_y} \right)^{-1} \beta^{-2} \int_{|q_x| \ll k_x} 4\pi \left[\left| V_{\vec{k},\vec{q},-\vec{k}-\vec{q}} \right|^2 n_{\vec{q}} \right]_{q=(q_x,-2k_y)} q_x^2 dq_x, \tag{7}$$

$$\beta = \frac{3k_y^2 - 3k_x^2 + 3k_y^4 - 6k_x^2 k_y^2 - k_x^4}{2k_x k_y (1 + 4k_y^2)}.$$

Here operator $(\partial / \partial k_y.)_\phi$ denotes the k_y-derivative under a constant value of function ϕ, the latter being a solution of the following equation:

$$\beta \frac{\partial \phi}{\partial k_x} - \frac{\partial \phi}{\partial k_y} = 0. \tag{8}$$

The integration in formula (4) is carried out over the small (compared to unity) values p_y, while in formula (7) the value q_x must be less than k_x, but may be of order one or even greater. Therefore, even under small values of ratios n_q/n_p (n_q, n_p being characteristic values of turbulent spectra in the zonal flows' region and in the region of large scales respectively), the term I^1_{zf} can be greater than the term I^0_{ls}; in this case the redistribution of spectrum owing to the nonlocal interaction with zonal flows will be the dominant process, and in place of Eq.(1) one should write the following equation for the spectrum evolution:

$$\frac{\partial n_{\vec{k}}}{\partial t} = I^0_{zf} + I^1_{zf}, \tag{9}$$

with the quantities I^0_{zf}, I^1_{zf} being defined by Eqs.(3),(6).

Equation (9) describes one-dimensional diffusion in k-space along the lines of constant values of the function $\phi(\vec{k})$ satisfying Eq.(8). The important fact is, that this function appears to be the same as the spectral density of the new invariant found in the Ref.[5]. This fact emphasizes the significance of such invariant for the drift turbulence theory.

Contours of Eq.(8) solution are shown in Fig.1. There is an interesting resemblance of Fig.1 to a pattern on the surface of a mussel: the curves $\phi(\vec{k}) = const$

are closed; they are crossing the point $\vec{k} = (0,0)$ at the same incline (equal to $3^{1/2}$).

It should be noted, that due to one-dimensionality of both the redistribution owing to interaction with large scales and the one owing to nonlocal interaction with the turbulence of zonal flows we should retain contributions of each of these processes if we want to take into account two-dimensionality of total redistribution whatever the process is dominant.

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