Communication through plasma sheaths via Raman (three-wave) scattering process

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A mechanism of communication through plasma sheaths which surround vehicles traveling at hypersonic velocities is proposed. The idea is to place a source of an intensive, high frequency electromagnetic pump wave on the vehicle and to use the nonlinearity of the medium to create a backscattered Stokes wave through the interaction of the pump wave with the signal-carrying wave sent by a ground transmitter. Such an interaction will be mediated by the plasma oscillations generated in the process of the resonant absorption of the signal-carrying wave. The advantages of this mechanism for information transfer are that the high frequency pump wave only is present in a small region near the vehicle and that the signal wave can have relatively low frequency.

I. INTRODUCTION

The interaction of electromagnetic waves with a plasma layer can be both beneficial and detrimental. The reflectivity of the plasma layer is used to our advantage in short wave broadcasting. On the other hand, the plasma sheaths in the vicinity of reentry or other hypersonic velocity vehicles can cause severe communication problems because of signal reflection or/and attenuation. There have been many suggestions to overcome this problem, for example, using high frequency laser or electron beams as the signal carriers.[1,2] Whereas there are advantages in using certain proposed methods, none of them have been found to be cheap to realize. Among the reasons are the amount of power required at the (distant) source and the accuracy required to ensure the signal beam intersects the target. The latter is particularly important if electron beams are used because of the influence of the earth's magnetic field. Other attempts have focused on improving the transmission properties of the plasma sheath by modifying it with chemical additives[3–5,1,2] or strong magnetic fields.[6,1,2] These approaches have appeared to be fairly effective, although the price of plasma sheath modification is also high. A magnetic field requires a heavy and bulky cooling system. The consumption rate of the chemical additives is high because of the supersonic flow, and, in addition, the most effective chemicals such as freon are bad for the environment.

The goal of this paper is to explore novel and potentially cost effective alternatives, which take advantage of nonlinear properties of the medium, rather than modifying it or using a high power ground source. The main idea is to place on board the vehicle a transmitter to send an intense high frequency electromagnetic pump wave to the region of reflection of the signal wave. The nonlinear, three-wave interaction of the pump wave and the Langmuir oscillations produced in the region of the signal reflection will result in the excitation of a third wave, called a Stokes wave, which will travel back toward the vehicle and carry the message encoded on the signal wave with it; see Fig. 1. The nonlinear interaction of an electromagnetic wave with Langmuir oscillations resulting in the excitation of a Stokes electromagnetic wave is usually referred to as Raman scattering.[7–10]

Of course, the signal carried on the Stokes wave has to compete with background noise generated by plasma turbulence. However, Langmuir turbulence will be at the low level of the thermal fluctuations, because it does not have any source to drive it. We will estimate the effect of the Langmuir noise in Sec. V. On other hand, the low frequency acoustic turbulence will be stronger and possibly can modify the plasma profile and even reverse the direction of the concentration gradient. Nevertheless, these profile modifications will not affect the resonant absorption process if their scales are smaller than the plasma resonance width. Moreover, the effect of the larger scales of the profile disturbances is to change the amplitude of the Stokes wave rather than its frequency, because the acoustic waves are much lower in frequency than the Langmuir waves (i.e., the profile disturbances are 'frozen'). Therefore, to filter out the low frequency acoustic noise, one should use a narrow band receiver tuned at the Stokes wave frequency.

Whereas the main aim of the present paper is to study stationary Raman scattering process, a sequel paper will describe how the nonstationary signal/pump wave interactions can give rise to several additional communication channels, including stimulating Brillouin scattering and acoustic pulses generated by a nonstationary radiative forcing. In that work, we will also discuss the use and advantages of a pulsed, rather than continuous, signal wave. The presence of several parallel and independent channels and the use of signal pulse trains greatly reduce the constraints on signal to noise ratio for effective communications. A brief preview of this material is given in Sec. V of this paper.

Raman scattering has been intensively studied in the physics of laser generated plasmas. In that context, the process can be considered as a convective instability of the pump wave resulting in the excitation of both the electromagnetic Stokes wave and plasmons. The latter cause the anomalous heating of plasma electrons.[7,9] The threshold value of the amplitude of the pump wave E_{pm} for such an instability is, according to Rosenbluth's criterion,[10]

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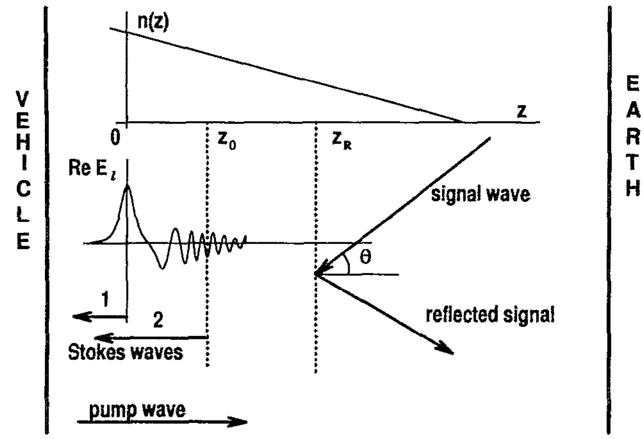

FIG. 1. Raman scattering of a pump wave on Langmuir oscillations induced by the signal wave. 1—Stokes wave generated in the resonant region $z \approx 0(\omega_p(0)=\omega_0)$, 2—Stokes (or anti-Stokes) wave excited in the wave region $z \approx z_0$, $k_{s,a}=k_{pm}-k_l(z_0)(\omega_{s,a}=\omega_{pm}\pm\omega_0)$. z_R is the point of signal wave reflection $(\omega_0=\omega_p(z_R)/\cos\theta)$.

$$E_{pm} > \frac{m\omega_0}{ec}\sqrt{\frac{2}{k_0 L}} \tag{1}$$

where ω_{pm}, k_{pm} are the frequency and wave vector of the pump wave, e, m are the charge and the mass of electron. For supercritical values of E_{pm}, an initial plasma wave perturbation will be amplified by a factor $\exp(2\pi)$ and the pump wave will be depleted by generating the Stokes and plasma waves. However, for our purposes we can operate the pump wave at subcritical values and this means that we may disregard the change in the plasma wave amplitude and consider it to be of a fixed value determined only by the process of the signal wave resonant absorption. From the mathematical point of view, this implies that the amplitude of excited Stokes wave can be found as a solution of a linear nonhomogeneous equation whose forcing is defined by the amplitudes of the pump and Langmuir oscillations.

There are two distinct regions where the plasma oscillations give appreciable contribution to Raman scattering, see Fig. 1. First the pump wave will interact with the large amplitude plasma oscillations at the resonance layer, at which $\omega_0 \approx \omega_p$, where the frequency of the signal wave,

$$\omega_p \equiv \omega_p(z) = \sqrt{4\pi e^2 n(z)/m} \tag{2}$$

is the plasma frequency in the sheath, $n(z)$ is the plasma density profile. Whereas the amplitude in this region is large, the wave number spectrum is broad, and only a small resonant part of the spectrum will contribute significantly to the reflected wave. The Raman scattering on the resonant layer is considered in Sec. III. Second, the pump wave will interact with the outwardly propagating plasma wave produced by the resonance layer. Because of the decreasing of plasma density this wave will have a slowly varying wave vector, and there will be a region in which there is a strong, three-wave, resonant interaction between the pump and signal induced plasma wave. Such interaction will also produce a Stokes wave which will return to the vehicle. This process is considered in Sec. IV.

Both processes, the Raman scattering of the pump wave on the resonant Langmuir oscillations and the scattering between the pump and the traveling plasma wave give the same order of magnitude contribution to the Stokes wave excitation. According to Secs. III and IV the ratio of the amplitudes of the Stokes $\mathbf{E}_s$ wave when it returns to the vehicle and the amplitude of the signal wave $\mathbf{E}_t$ is given by the following formula:

$$\frac{|\mathbf{E}_s|}{|\mathbf{E}_t|} \approx \frac{eL|\mathbf{E}_{pm}|}{mc^2}(k_0 L)^{-1/3}. \tag{3}$$

For a pump field of $\alpha 10^5$ V/m, a plasma density decay length and sheath thickness of β (m) and signal wave number of γ (m^{-1})

$$\frac{|\mathbf{E}_s|}{|\mathbf{E}_t|} \approx \frac{2}{9}\,\alpha\beta(\gamma\beta)^{-1/3}.$$

Because of the cube root, the factor $(\gamma\beta)^{-1/3}$ is of order one. Therefore the significant factor in the fraction is the strength of the pump field and the plasma sheath thickness. If we choose the former to be the maximum value for which no appreciable further ionization of the plasma occurs, then $\alpha \approx 1/2$. Estimates on the thickness of the layer vary from estimates based on the distance of the bow shock from the vehicle wall to estimates based on a model for the ambipolar diffusion of the plasma through the bow shock. A reasonable value for β should be between 1/10 and 1, so that the ratio is anywhere between 1% to 10%.

What is remarkable is that the result (3) is independent of the pump and plasma frequency as long as the former is greater than the latter. There are other nonlinearities that also generate Stokes waves which depend on the ratio ω_p/ω_{pm}, but they are smaller than the contribution of the term which gives rise to (3) (see the formulas for the general case given at the end of Secs. III and IV). The strong dependence of the result on the thickness of the plasma sheath is a direct reflection of the importance of the thickness of the resonant layer or the distance over which the three-wave scattering process takes place. The fraction (3) can be also be interpreted as the work done by the pump field in moving an electron a distance of the plasma sheath thickness in units where mc^2 is one. Further, nonlinear process is reversible. A message encoded on an outward traveling Stokes wave can be transferred to a signal wave at any desired frequency through interaction with a pump wave. This fact is briefly discussed in the conclusion of this paper, Sec. V.

II. RESONANT ABSORPTION OF THE SIGNAL WAVE

To investigate the nonlinear excitation of the Stokes wave, one must know the structure of the wave field excited by the signal electromagnetic wave. This is the classical and well understood problem of the wave propagation and reflection of an electromagnetic wave in an inhomogeneous plasma layer, usually referred to as resonant absorption.[11-13] We will briefly summarize the main results.

Let us assume that a plane electromagnetic signal wave

$$\mathbf{E}_t=\hat{e}(A_t \exp(i\mathbf{k}_0\mathbf{r}-i\omega_0 t)+\text{c.c.}), \tag{4}$$

with $\omega_0=c|\mathbf{k}_0|,\mathbf{k}_0=-k_0(\sin\theta,0,\cos\theta)$, $\hat{e}=(0,1,0)$, propagates toward a plasma layer with a linear density profile:

$$n(z)=n_0(1-z/L). \tag{5}$$

The incoming signal wave is reflected at

$$z=z_R \tag{6}$$

where

$$\omega=\omega_p(z_R)/\cos\theta. \tag{7}$$

However it also generates strong plasma oscillations at the value of z (which we take to be $z=0$) where

$$\omega_0=\omega_p(0)=(4\pi e^2 n_0/m)^{1/2}. \tag{8}$$

The expression for the longitudinal (i.e., z) component $E_l(z)$ of the electric field, corresponding to plasma oscillations, is[11,13]

$$E_l(z)=-i\pi\lambda \sin\theta\Lambda A_t \exp(-i\omega_0 t)$$
$$\times[\text{Gi}(-p)-i\text{Ai}(-p)]+\text{c.c.}, \tag{9}$$

where

$$p=\lambda\left(\frac{z}{L}+i\frac{\nu}{\omega_0}\right), \tag{10}$$

$$\lambda=(k_0L)^{2/3}\left(\frac{mc^2}{3T}\right)^{1/3}, \tag{11}$$

ν is the frequency of electron collisions, T is the temperature, k_0,ω_0,E_t are the wave vector, frequency and the intensity of the signal wave in vacuum ($\omega_0=ck_0$), Gi and Ai are Airy functions, and c.c. refers to complex conjugate.

The absolute value of Λ measures the fraction of the signal wave electric field which reaches the resonant point $z=0$ (see Ref. 13) and is given by

$$\Lambda=-\text{Ai}'(q)/[1-iq\pi^2\text{Ai}'(q)(\text{Bi}'(-q)+i\,\text{Ai}'(-q))], \tag{12}$$

where

$$q=(k_0L)^{2/3}\sin^2\theta;$$

and the prime denotes differentiation with respect to the argument. There exists an optimal angle θ for which the excitation of Langmuir oscillations is maximal;[11–13] for this angle

$$q\approx\tfrac{1}{2},\ \sin^2\theta\approx(k_0L)^{-2/3}/2,\quad |\Lambda|\approx\tfrac{1}{4}. \tag{13}$$

Note, in order that the result does not depend sensitively on the angle θ, it is best to choose k_0 such that $k_0L\approx 1$. In the case of cold plasma,

$$\frac{\lambda\nu}{\omega_p}=\frac{\nu}{\omega_p}(k_0L)^{2/3}\left(\frac{mc^2}{3T}\right)^{1/3}\gg 1, \tag{14}$$

the expression (9) can be simplified to

$$E_l(z)=-i\pi\lambda\sin\theta\Lambda A_t\exp(-i\omega_0 t)\left[-\frac{1}{\pi p}\right.$$
$$\left.+\pi^{-1/2}p^{-1/4}\exp\left(\frac{2i}{3}p^{3/2}+\frac{i\pi}{4}\right)\right]+\text{c.c.} \tag{15}$$

This formula is interesting because it points out that the signal induced plasma electric field has one stationary component (corresponding to the first term in the brackets) centered at the resonance layer, and one propagating component (the second term) whose local wave number is $k_l=\lambda p^{1/2}/L$ and which propagates in the positive z direction away from $z=0$, see Fig. 1. If condition Eq. (13) is not satisfied we can still use formula (15) for qualitative analysis, with collision frequency ν replaced by (see Ref. 11)

$$\nu_{\text{eff}}=\omega_p(k_0L)^{-2/3}\left(\frac{3T}{mc^2}\right)^{1/3}.$$

As discussed, the Raman scattering of the pump wave on Langmuir oscillations at the resonant layer and wave domain are different. Oscillations in the resonant region have large amplitude, but broad spectrum in k-space, so that only a small part of the spectrum is in resonance with the pump and Stokes waves. On other hand, oscillations in the wave region are almost monochromatic (with slowly varying wave vector), while their amplitude is small in comparison with the one in the resonant region. The excitation of the Stokes wave by nonlinear interaction of an electromagnetic pump wave with the longitudinal plasma oscillations in the resonant layer is considered in Sec. II. Section III is devoted to a study of the generation of the Stokes wave by the interaction of the pump wave and a Langmuir wave with slowly varying in space wave vector. Notice, that direct resonant interaction of the pump and the signal (electromagnetic) waves is impossible, because the second derivative of the frequency of electromagnetic wave with respect to wave vector is positive.[14]

III. REFLECTION OF THE PUMP WAVE FROM THE RESONANT LAYER

Consider a pump electromagnetic wave with electric field $\mathbf{E}_{pm}$, wave vector $\mathbf{k}_{pm}$, and frequency ω_{pm} propagating in the plasma sheath from the direction of the vehicle toward the region of resonant absorption and reflection of the signal wave. First, suppose for simplicity that the pump wave propagates exactly in z direction with a frequency $\omega_{pm}\gg\omega_p$, see Fig. 1. We will give the expressions for an arbitrary angle of pump wave propagation and values of ω_{pm} comparable to ω_p at the end of this section.

The equation for the electric field in the plasma sheath is

$$\frac{\partial^2\mathbf{E}}{\partial t^2}+c^2\nabla\times\nabla\times\mathbf{E}=-4\pi\frac{\partial\mathbf{j}}{\partial t}, \tag{16}$$

where $\mathbf{j}$ is the current density, which can be found from the equation for electron motion

$$n\left(\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}\right)=-\nabla p/m-(en/m)(\mathbf{E}+\mathbf{v}\times\mathbf{B}/c), \tag{17}$$

and the continuity equation for the electron liquid:

$$\frac{\partial n}{\partial t}+\mathrm{div}(n\mathbf{v})=0. \qquad (18)$$

It can be easily shown that the main contributions from the nonlinear terms in Eqs. (17) and (18), important in the limit $\omega_{pm}\gg\omega_p$, come from the product containing the perturbation in the plasma density $\tilde{n}$ and the electric field of Langmuir oscillations [the second term on the right-hand side of (17)]. Hence, Eqs. (17) and (18) give us the following expression for the time derivative of $\mathbf{j}$:

$$\frac{\partial \mathbf{j}}{\partial t}=-\frac{\partial}{\partial t}\, e\, n\, \mathbf{v}=\frac{\omega_p^2}{4\pi}(1+\tilde{n}/n)\mathbf{E}. \qquad (19)$$

Substituting this expression into Eq. (16) and performing Fourier transformation over the z variable, we get

$$\frac{\partial^2\hat{E}}{\partial t^2}+\mu\,\frac{\partial\hat{E}}{\partial t}+(c^2k^2+\omega_p^2)\hat{E}-\frac{i\omega_0^2}{L}\,\frac{\partial\hat{E}}{\partial k}$$
$$=-\frac{\omega_p^2}{n}\int n(k_l)\hat{E}_{pm}(\tilde{k})\,\delta(k-k_l-\tilde{k})\,dk_l d\tilde{k}, \qquad (20)$$

where $\hat{E},\hat{E}_{pm}$ are the Fourier transforms of the electric field amplitudes of the scattered and pump waves respectively, and $n(k)$ is the Fourier transform of the plasma density associated with Langmuir oscillations. In deriving (20) we took into account that the profile of the plasma density (and, hence, ω_p^2) is linear [see Eq. (5)], and included a small amount of damping μ in order to identify the correct contours of integrations in wave-number space.

Supposing the pump wave to be monochromatic,

$$E_{pm}=A_{pm}\exp(-i\omega_0 t+ik_{pm}z)+\text{c.c.},\quad \text{i.e.},$$
$$\hat{E}_{pm}(k)=A_{pm}e^{-i\omega_{pm}t}\delta(k-k_{pm})+A_{pm}^*e^{i\omega_{pm}t}\delta(k+k_{pm}), \qquad (21)$$
$$A_{pm}=\text{const},\quad \omega_{pm}=k_{pm}c, \qquad (22)$$

and expressing $n(k)$ in terms of longitudinal electric field [see Eq. (9)] according to the Gauss theorem:

$$n(k)=-\frac{ik}{4\pi e}\,\hat{E}_l(k), \qquad (23)$$

one can write the right-hand side of Eq. (20) as follows:

$$\frac{ie}{m}[(k-k_{pm})\hat{E}_l(k-k_{pm})A_{pm}e^{-i\omega_{pm}t}+(k+k_{pm})$$
$$\times\hat{E}_l(k+k_{pm})A_{pm}^*e^{i\omega_{pm}t}],$$

where $\hat{E}_l(k)$ is the Fourier transform of the longitudinal electric field (9):

$$\hat{E}_l=\frac{1}{2\pi}\int_{-\infty}^{\infty}E_l(\mathbf{r})dz.$$

Using formulas (9) and (4) we can write $\hat{E}_l$ in the following form:

$$\hat{E}_l(k)=-i\pi\sin\theta(\Lambda\mathscr{E}_kA_t\exp(-i\omega_0 t)$$
$$-\Lambda^*\mathscr{E}_{-k}^*A_t^*\exp(i\omega_0 t)),$$

where $\mathscr{E}_k$ is the Fourier transform of

$$\lambda[\mathrm{Gi}(-p)-i\,\mathrm{Ai}(-p)]. \qquad (24)$$

Therefore, the right-hand side of Eq. (20) is

$$\frac{\pi e}{m}\sin\theta[A_tA_{pm}(k-k_{pm})\Lambda\mathscr{E}_{k-k_{pm}}\exp(-i(\omega_0+\omega_{pm})t)$$
$$+A_t^*A_{pm}^*(k+k_{pm})\Lambda^*\mathscr{E}_{-k-k_{pm}}\exp(i(\omega_0+\omega_{pm})t)$$
$$+A_tA_{pm}^*(k+k_{pm})\Lambda\mathscr{E}_{k+k_{pm}}\exp(-i(\omega_0-\omega_{pm})t)$$
$$+A_t^*A_{pm}(k-k_{pm})\Lambda^*\mathscr{E}_{-k+k_{pm}}\exp(i(\omega_0-\omega_{pm})t)]. \qquad (25)$$

The solution of Eq. (20) corresponding to the Stokes wave is

$$\hat{E}_s=A_se^{-i\omega_st}+A_s^*e^{i\omega_st}(\omega_s=\omega_{pm}-\omega_0), \qquad (26)$$

and the anti-Stokes wave is given by

$$\hat{E}_a=A_ae^{-i\omega_at}+A_a^*e^{i\omega_at}(\omega_a=\omega_{pm}+\omega_0). \qquad (27)$$

Substituting Eqs. (26) and (27) into Eq. (20) we get the following equations for A_s,A_a:

$$(\omega_0^2+c^2k^2-\omega_s^2-i\omega_s\mu)A_s-\frac{i\omega_0^2}{L}\,\frac{\partial A_s}{\partial k}$$
$$=\frac{ie}{m}(k-k_{pm})\Lambda^*\mathscr{E}_{-k+k_{pm}}^*A_t^*A_{pm}, \qquad (28)$$
$$(\omega_0^2+c^2k^2-\omega_a^2-i\omega_a\mu)A_a-\frac{i\omega_0^2}{L}\,\frac{\partial A_a}{\partial k}$$
$$=\frac{ie}{m}(k-k_{pm})\Lambda\mathscr{E}_{k-k_{pm}}A_tA_{pm}. \qquad (29)$$

It is not hard to see that the (inverse) Fourier transforms of A_s,A_a will be dominated by values of k near k_s,k_a which are zeros of

$$\omega_0^2+c^2k^2-\omega_s^2-i\omega_s\mu,$$
$$\omega_0^2+c^2k^2-\omega_a^2-i\omega_a\mu,$$

namely

$$\pm k_{s,a}=\pm\frac{1}{c}\sqrt{\omega_{s,a}^2-\omega_0^2}\left(1+\frac{i\mu\omega_{s,a}}{2(\omega_{s,a}^2-\omega_0^2)}\right). \qquad (30)$$

Indeed the spread of wave numbers $|k-k_{s,a}|$ which will contribute is of order $(k_0L)^{-1/2}\omega_0/\omega_{pm}$ which is assumed to be small. Therefore, to leading order, we can calculate $A_{s,a}$ by neglecting $(1/L)(\partial A_{s,a}/\partial k)$ and solving Eqs. (28) and (29) algebraically. Furthermore, when damping μ is introduced, the positive zero $k_{s,a}$ is in the upper half k-plane and the negative zero $-k_{s,a}$ is in the lower half plane.

If we are interested in the region $z<0$ on the vehicle side of the resonant layer, the scattered field in coordinate space is

$$E_{s,a}(z,t)=\int_{-\infty}^{\infty}\hat{E}_{s,a}(k,t)\exp(ikz)dk, \qquad (31)$$

and can be calculated by closing the contour in the lower half plane. Hence, we will get the main contribution in this case from the domain adjacent to wave vector $-k_{s,a}$. By solving Eqs. (28) and (29) algebraically and substituting the values A_s, A_a obtained in (31), we get the following expressions for the Stokes and anti-Stokes electric fields in real space for $z<0$:

$$E_s = -\frac{2\pi^2 e}{mc^2} \sin\theta \Lambda^* \mathscr{E}^*_{k_{pm}-k_s} A_t{}^* A_{pm} \times \exp(-i\omega_s t - ik_s z) + \text{c.c.}, \tag{32}$$

$$E_a = -\frac{2\pi^2 e}{mc^2} \sin\theta \Lambda \mathscr{E}_{-k_{pm}+k_a} A_t A_{pm} \times \exp(-i\omega_a t - ik_a z) + \text{c.c.}, \tag{33}$$

where [taking into account $\omega_{pm} \gg \omega_0$, and Eq. (30)]

$$k_s \approx k_a \approx \omega_{pm}/c.$$

According to Eqs. (32) and (33) the dominant wave number in the shape of $E_l(z,t)$ is $k_l = k_{pm} + k_{s,a} \approx 2k_{pm}$. Remembering also that $\omega_{a,s} = \omega_{pm} \pm \omega_0$, one can view the main contribution as coming from the three-wave interaction.

Now we have to substitute into Eqs. (32) and (33) the value of $\mathscr{E}_{k_l}$ [see Eq. (24)] corresponding to the plasma oscillations at the resonant layer, i.e., to the first term in the square brackets of (15):

$$\mathscr{E}_k = \frac{iL}{\pi} H(k) \exp(-k_l L \nu/\omega_0), \tag{34}$$

where $H(k)$ is the Heaviside function. This means that there is no anti-Stokes wave excited at the region $z<0$ in the process of scattering on the resonant layer. For the Stokes field, on the other hand, we get the following expression:

$$E_s = \frac{2\pi e L}{mc^2} \Lambda^* \sin\theta \exp(-k_l L \nu/\omega_0) A_t^* A_{pm} \times \exp(-i\omega_s t - ik_s z) + \text{c.c.}, \tag{35}$$

which leads to the estimate (3) if we substitute the value of Λ (13) corresponding to the optimal angle of signal wave incidence, and put $\exp(-k_l L \nu/\omega_0) \approx 1$.

It is interesting that on the low density side of the plasma (i.e., on the side away from the vehicle) there is no Stokes wave, while the amplitude of the anti-Stokes wave is of the order of ω_0/ω_{pm} smaller than the amplitude of backscattered Stokes wave (35). This can be explained by the fact that, from the resonance conditions, the forward scattering is proportional to the low-wave-number component of the density disturbance, which is about ω_0/ω_{pm} times weaker than the short-scale density harmonics corresponding to $k_l = 2k_{pm}$.

Now, for reference, we will write an expression for the Stokes wave electric field in the general case, i.e., for values of ω_{pm} comparable to ω_p, ω_0 and arbitrary angle of incidence and polarization of the pump wave (see Fig. 2),

$$E_s = A_s \exp(-i\omega_s t + i\mathbf{k}_s \mathbf{r}) + \text{c.c.}, \tag{36}$$

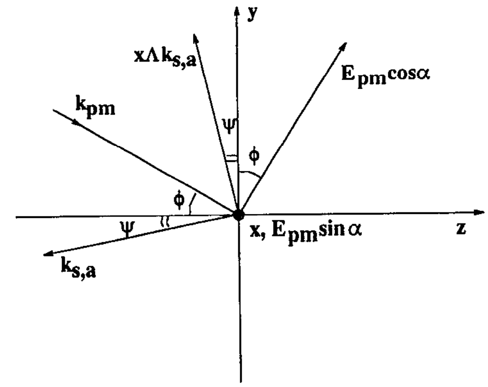

FIG. 2. Geometry of the problem in the case of arbitrary angles of incidence ϕ and polarization α of the pump wave. ψ is the angle of the scattered (Stokes or anti-Stokes) wave propagation.

$$A_s = \frac{4\pi^2 eL}{mc^2 k_{sz}} \sin\theta \Lambda^* A_{pm} A_t^* \Bigg(\sin^2\alpha \Bigg[\left(1 + \frac{\omega_0}{\omega_{pm}}\right) k_{lz} - 2\sin^2\frac{\phi}{2} \frac{\omega_0}{\omega_{pm}} k_{pm} \Bigg]^2 + \cos^2\alpha \Bigg[\left(k_{lz}\left(1 + \frac{\omega_0}{\omega_{pm}}\right) + \frac{\omega_0}{\omega_{pm}} k_{pm} \cos\phi \right) \cos(\phi+\psi) - k_{lz} \frac{\omega_0}{\omega_{pm}} \times \sin\phi \sin\psi - \frac{\omega_0}{\omega_{pm}} k_{pm} \cos\psi \Bigg]^2 \Bigg)^{1/2}, \tag{37}$$

where

$$\mathbf{k}_s = (0, k_{sy}, k_{sz}),$$

$$k_{sy} = k_{pm} \cos\phi,$$

$$k_{sz} = \sqrt{(\omega_{pm}^2 - 2\omega_0 \omega_{pm})/c - k_{sy}^2}$$

is the wave number of the backscattered Stokes wave, and

$$\omega_s = \sqrt{c^2 k_s^2 + \omega_p^2} \tag{38}$$

is its frequency; ϕ, α are the angles of incidence and polarization of the pump wave (see Fig. 2), ψ is the angle of propagation of the Stokes wave:

$$\psi = \arctan\frac{k_{sy}}{k_{sz}},$$

$k_l = (0, 0, k_{sz} - k_{pm\,z})$ is the resonant wave number of Langmuir oscillations.

IV. RAMAN SCATTERING ON A PLASMA WAVE WITH SLOWLY VARYING WAVE VECTOR

After passing the resonant region, the pump wave propagates through the domain occupied by the plasma wave [described by the second term in square brackets of Eq. (15)]. The wave vector of such plasma wave increases with the decrease of plasma concentration, so that, in the vicinity of some point z_0, the nonlinear interaction of the pump and plasma waves becomes resonant (see Fig. 1):

$$\omega_s = \omega_{pm} - \omega_0, \tag{39}$$

$$\mathbf{k}_l(z_0)=\mathbf{k}_s(z_0)-\mathbf{k}_{pm}(z_0) \tag{40}$$

for the Stokes wave, or

$$\omega_a=\omega_{pm}+\omega_0, \tag{41}$$

$$\mathbf{k}_l(z_0)=\mathbf{k}_a(z_0)-\mathbf{k}_{pm}(z_0) \tag{42}$$

for the anti-Stokes wave.

This situation is similar to the problem of the nonlinear interaction of waves which are in resonance for a limited period of time considered by Ablowitz, Funk, and Newell.[15] The present problem is simpler, however, in a sense that only a small fraction of the energy of the pump and plasma waves is converted to the Stokes wave and one can disregard the change of their amplitude in the process of the nonlinear interaction. Therefore, the generation of the Stokes and anti-Stokes waves can be again described by the linear (28) and (29) with the source terms proportional to the product of amplitudes of the pump and plasma waves. The only difference is that, instead of Eq. (34), one should use the Fourier transform of the second part of the expression (15) corresponding to a traveling plasma wave. Certainly, the value of the plasma frequency should be taken now in the point z_0 determined by the conditions (39) and (40) or (41) and (42).

To find an expression for the spectrum of plasma wave, it is useful first to expand the second part of Eq. (15) in powers of the parameter $(z-z_0)/z_0$, taking into account that the resonant interaction takes place only in a small vicinity of the point z_0:

$$E_l(z)\approx -i\pi^{1/2}\lambda p_0^{-1/4}\sin\theta\Lambda E_t \times\exp\left(\frac{2i}{3}p_0^{3/2}+ip_0^{3/2}\frac{(z-z_0)}{z_0}+\frac{i}{4}p_0^{3/2}\frac{(z-z_0)^2}{z_0^2}+i\pi/4\right)+\text{c.c.}, \tag{43}$$

where

$$p_0=\lambda z_0/L. \tag{44}$$

The Fourier transform of the expression (43) can be easily found; as a result we get the following expression for $\mathcal{E}_k$:

$$\mathcal{E}_k=2\pi^{-1/2}(i-1)L\exp(-ip_0^{3/2}/3+i\pi/4+ik_zz_0-ik_z^2z_0^2p_0^{-3/2}). \tag{45}$$

Note that both Stokes and anti-Stokes backscattered waves are excited in this case. Substituting Eq. (45) into Eqs. (32) and (33) we get for their electric fields:

$$E_s=-\frac{4\pi^{3/2}eL}{mc^2}\sin\theta\Lambda^*(1+i)A_t^*A_{pm}\times\exp(ip_0^{3/2}/3-i\pi/4+ik_lz_0+ik_l^2z_0^2p_0^{-3/2}-i\omega_st-ik_sz)+\text{c.c.}, \tag{46}$$

$$E_a=-\frac{4\pi^{3/2}eL}{mc^2}\sin\theta\Lambda(1-i)A_tA_{pm}\times\exp(-ip_0^{3/2}/3+i\pi/4+ik_lz_0-ik_l^2z_0^2p_0^{-3/2}-i\omega_at-ik_az)+\text{c.c.}, \tag{47}$$

where $k_l\approx 2k_{pm}$ is the value of plasma wave number at the region of three-wave resonance.

Taking into account the dispersion law for plasma waves

$$\omega_l^2=\omega_p^2+\frac{3T}{m}k_l^2 \quad (\omega_p(0)=\omega_0), \tag{48}$$

and from formulas (2) and (5) for the plasma frequency and the density profile, one can easily find an expression for the point z_0 in terms of k_l:

$$z_0=\frac{3TL}{m\omega_0^2}k_l^2. \tag{49}$$

One can see that expressions (46) and (47) give the same order of magnitude for the Stokes wave amplitude as in the case of scattering on the resonant layer. Substitution of the optimal value of Λ [see (13)] again results in the estimate (3).

Now let us write down expressions for E_s, E_a for arbitrary angles of incidence and polarization of the pump wave and its frequencies comparable to the plasma frequency:

$$E_s=-\frac{4\pi^{3/2}eL}{mc^2k_{sz}}\sin\theta\Lambda^*(1+i)A_t^*A_{pm}\exp(ip_0^{3/2}/3-i\pi/4+ik_lz_0+ik_l^2z_0^2p_0^{-3/2}-i\omega_st-i\mathbf{k}_s\mathbf{r}) \times\Bigg(\sin^2\alpha\left[\left(1+\frac{\omega_p}{\omega_{pm}}\right)k_l-2\sin^2\frac{\phi}{2}\frac{\omega_p}{\omega_{pm}}k_{pm}\right]^2+\cos^2\alpha\Bigg[\left(k_l\left(1+\frac{\omega_p}{\omega_{pm}}\right)+\frac{\omega_p}{\omega_{pm}}k_{pm}\cos\phi\right) \times\cos(\phi+\psi_s)-k_l\frac{\omega_p}{\omega_{pm}}\sin\phi\sin\psi_s-\frac{\omega_p}{\omega_{pm}}k_{pm}\cos\psi_s\Bigg]^2\Bigg)^{1/2}+\text{c.c.}, \tag{50}$$

$$E_a=-\frac{4\pi^{3/2}eL}{mc^2k_{az}}\sin\theta\Lambda(1-i)A_tA_{pm}\exp(-ip_0^{3/2}/3+i\pi/4+ik_lz_0-ik_l^2z_0^2p_0^{-3/2}-i\omega_at-i\mathbf{k}_a\mathbf{r}) \times\Bigg(\sin^2\alpha\left[\left(1+\frac{\omega_p}{\omega_{pm}}\right)k_l-2\sin^2\frac{\phi}{2}\frac{\omega_p}{\omega_{pm}}k_{pm}\right]^2+\cos^2\alpha\Bigg[\left(k_l\left(1+\frac{\omega_p}{\omega_{pm}}\right)+\frac{\omega_p}{\omega_{pm}}k_{pm}\cos\phi\right) \times\cos(\phi+\psi_a)-k_l\frac{\omega_p}{\omega_{pm}}\sin\phi\sin\psi_a-\frac{\omega_p}{\omega_{pm}}k_{pm}\cos\psi_a\Bigg]^2\Bigg)^{1/2}+\text{c.c.}, \tag{51}$$

where

$$\mathbf{k}_{a,s}=(0,k_{a,s\,y},k_{a,s\,z}),$$

$$k_{a,s\,y}=k_{pm}\cos\phi,$$

$$k_{a,s\,z}=\sqrt{(\omega_{pm}^2\pm 2\omega_0\omega_{pm})/c-k_{a,s\,y}^2},$$

$$\omega_{a,s}=\sqrt{c^2k_{a,s}^2+\omega_p^2},$$

$$\psi=\arctan\frac{k_{a,s\,y}}{k_{a,s\,z}},$$

and ω_p should be taken at the point z_0 (49) according to (2) and (5). The value of the resonant wave number of Langmuir wave is defined as follows:

$$k_l=k_{pm}\cos\phi+\sqrt{k_{pm}^2\cos^2\phi\pm 2\omega_{pm}\omega_0/c^2+\omega_0^2/c^2}, \tag{52}$$

where the sign '+' corresponds to the anti-Stokes wave and the sign '−' to the Stokes wave.

V. CONCLUSION

We have shown in this paper that the process of the Raman scattering of a pump electromagnetic wave on Langmuir oscillations produced by the signal wave can be effective as a mechanism of information transfer *to the vehicle* during the reentry 'blackout' of radio communications. Of course, we have used an idealized model based on a simple slab geometry of the plasma sheath and have disregarded all the complexity of various physical chemical processes inside the sheath. A study taking account for a real plasma sheath geometry and possible extra ionization due to the pump electromagnetic field is underway, with the aid of computer simulations.

We would like to point out that the process of the excitation of the Stokes wave is reversible. One can produce a plasma wave by the nonlinear interaction of two high frequency electromagnetic waves.[16,17] This process will have the same amplitude as the process of Stokes wave generation by scattering on the plasma wave considered in this paper. Further, the excited plasma wave will travel toward the high concentration region of the plasma sheath until reaching the resonant point $\omega=\omega_p$. Afterwards, this plasma wave will be reflected and an electromagnetic wave will be emitted. The latter process has been shown to have the same amplitude as the process of resonant absorption by Hinkel-Lipsker, Fried, and Morales.[13] The excited electromagnetic wave will have a frequency equal to the difference of frequencies of the two pump electromagnetic waves. This frequency may be much less that the frequencies of the pump waves, and less than the maximal plasma frequency of the sheath. Such a process may be useful for the sending the signals *from* reentry vehicles, because high frequency electromagnetic waves are subject to absorption in the atmosphere due to the oxygen and water vapor,[1,2] and the use of low frequencies may be helpful.

Another mechanism which could be used for sending information from the reentry vehicle can be based on the Brillouin scattering[18] of an electromagnetic wave sent by a ground transmitter on the sound wave generated on the vehicle and carrying the message through the plasma sheath. The advantage of this mechanism is that the sound is not as sensitive to the changes in the plasma concentration as the Langmuir wave, though it is sensitive to the changes in the electron temperature.

Let us briefly discuss sensitivity of the Raman signal to the noise generated by the background turbulence. One should understand that Langmuir turbulence will be at the level of thermal fluctuations which is relatively low. A signal wave produced by a ground transmitter with a power of 10 MW will generate Langmuir oscillations with an electric field amplitude some 50 times stronger than the electric field of the Langmuir thermal noise at $T\approx 1$ eV. For lower transmitter powers, the issue of reducing the signal to noise ratio may arise.

In a sequel paper in preparation we have studied: (1) Raman scattering by a train of discrete pulses of the signal wave; (2) stimulated Brillouin scattering process; (3) acoustic wave generation by a nonstationary radiative forcing associated with the pulse trains. Each of these processes can be exploited as an independent communication channel in a parallel scheme for information transfer. The more channels of communication that are available, the less will be the required signal to noise ratio for effective interpretation of the signal.[19] We will analyze also the achievable rate of information transmission via the above mentioned channels (this rate depends on the relaxation time of the medium).

Finally, an experimental study of the nonlinear three-wave interactions for the purposes of communication through plasma sheaths would be very helpful. In particular, the real situation could be modeled in laboratory experiments on the interaction of the laser radiation with laser-produced plasma using two laser beams with different frequencies for the modeling of the signal and pump waves.

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