(K)inetic mechanisms for the growth of drift-ballooning modes

A. B. Mikhailovskii, S. V. Nazarenko, and A. P. Churikov

I. V. Kurchatov Institute of Atomic Energy, Moscow

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Kinetic mechanisms for the growth of drift-ballooning modes are studied theoretically. It is found that these modes may be unstable as a result of their resonant and collisional interactions with ions. The instability growth rates depend on the relation between the temperature gradient and the density gradient.

INTRODUCTION

It follows from single-fluid MHD with an oblique ...gnetic) viscosity that as the toroidal wave num... r is increased unstable ideal ballooning modes ...come stable drift-ballooning modes, because of a ...bilizing effect of the finite ion Larmor radius ...f. 1, for example). From the standpoint of ...etic theory, the meaning is that only the hydro...gnetic stability of the drift-ballooning modes is ...olved here. In this connection it is important ... determine whether a kinetic growth of these ...des is possible by virtue of their resonant or ...llisional interaction with plasma particles. That ...estion is the subject of the present paper.

Since the drift-ballooning modes are associated ...th Alfvén wave branches, the instabilities with ...ich we are concerned here may be thought of as ...rtain versions of the class of kinetic Alfven in...bilities of a tokamak plasma. The study of the ...stabilities of this class was begun in Ref. 2, ...ere an analysis was made of the resonant inter...tion of local Alfvén waves modified by effects of ... magnetic-field curvature, with untrapped ions. ... analysis of Ref. 2 dealt with the case of per...rbations with $\varepsilon^{1/2}v_{Ti}/qR < \omega < v_{Ti}/qR$, where ω ... the wave frequency, $\varepsilon = a/R$ is the inverse as...ct ratio, a and R are respectively the minor and ...or radii of the torus, $v_{Ti} = \sqrt{2T_i/M_i}$ is the ion ...rmal velocity, T_i, and M_i are respectively the ...mperature and mass of an ion, and q is the safety ...tor. The condition $\omega > \varepsilon^{1/2}v_{Ti}/qR$ allows us to ...ore the weakening of the resonant interaction ...ich results from the trapping of particles in re...ons of weak magnetic field (on the outer side of ... torus). In Ref. 3, in contrast with Ref. 2, a ...udy was made of the resonant interaction of ...ydam perturbations stabilized by the effects of the ...ite ion Larmor radius, with untrapped ions, in ... opposite limit, $\omega < \varepsilon^{1/2}v_{Ti}/qR$, in which this ...pping of particles is extremely important. Also ... interest for the problem at hand is the collisional ...eraction of waves with untrapped ions under the ...ndition $\omega < \nu_i/\varepsilon$, where ν_i is the rate of ion collisions. According to Ref. 2, this process ca... described in terms of a longitudinal ion visco... calculated in the banana regime. The role pl... by the process in the ideal m = 1 kink modes ... the index of the azimuthal harmonic) was stu... in Ref. 2. Waves with $\omega > \nu_i/\varepsilon$ were also st... there.

In Sec. 2 we present the basic equation... derive a general dispersion relation for drift-...looning modes, incorporating the effects liste... This relation is analyzed in Sec. 3. Conclus... are drawn in Sec. 4.

2. INITIAL EQUATIONS

We start from the equation for ideal ball... modes, with drift effects and effects of the i... tion with ions (cf. Ref. 4, for example):

$$\frac{d}{dt}\left(t^2\frac{d\xi}{dt}\right)-U_0\xi+t^2\frac{\omega^2}{\omega_A^2}\left(1-\frac{\omega_{pi}^*}{\omega}\right)+\sigma=0.$$

Here $\omega_A = Sc_A/qR$ is the Alfvén frequency, ... $q'a/q$ is the shear, $U_0=4\varepsilon^2\beta_p\times(1-1/q^2+3\varepsilon^2\beta_p/2)/S$... magnetic well, β_p is the ratio of the pressur... the pressure of the poloidal magnetic field, ... the ion drift frequency in terms of the press... gradient, t = Sy, and y is the standard ball... variable. The function ξ is a Fourier compo... of the radial displacement of the plasma. Th...ity σ incorporates effects of the resonant an... lisional interactions of ions with waves and is... fined by

$$\sigma=-\frac{2t}{\rho_0R\omega_A^2}(\delta\tilde{p}_k\sin\theta)^{(0)},$$

where ρ_0 is the plasma density, $\delta\tilde{p}_k$ is a Fo... component of the oscillatory part of the pert... pressure, and $(\ldots)^{(0)}=\int_0^{2\pi}(\ldots)d\theta/2\pi$.

We write the perturbed pressure in the ...

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$$\delta\tilde{p}=\frac{M_i}{4}\int v^2\tilde{f}\,d\mathbf{v}, \tag{3}$$

where $\tilde{f} = \hat{f}\exp(im\theta)$ is the perturbed part of the distribution function, and θ is the poloidal angle. To find the function $\hat{f}$ we use the drift kinetic equation (Ref. 2, for example)

$$-i\omega\hat{f}+\frac{v_\parallel}{qR}\frac{\partial\hat{f}}{\partial\theta}+i\omega\frac{M_i}{2T}v^2\left(1-\frac{\hat{\omega}^*}{\omega}\right)\frac{a}{mR}F\sin\theta\hat{\xi}'=\mathrm{St}(\hat{f}). \tag{4}$$

Here $v_\parallel$ is the velocity of the particles along the magnetic field, $\hat{\omega}^*=\omega_{ni}^*[1+\eta(M_iv^2/2T-{}^3/_2)]$, ω_{ni}^* is the drift frequency in terms of the pressure gradient, $\eta = \partial\ln T/\partial\ln n_0$, F is the equilibrium Maxwellian distribution function (normalized to the particle density n_0, as usual), St is the collision term, and the prime denotes a derivative with respect to the minor radius.

Introducing a function h, by analogy with Ref. 2,

$$\hat{f}=-ihF\left(1-\frac{\hat{\omega}^*}{\omega}\right)\frac{a}{mR}\hat{\xi}', \tag{5}$$

and using (4) and (5), we find an equation for h:

$$-i\omega h+\frac{v_\parallel}{qR}\frac{\partial h}{\partial\theta}-i\omega\frac{M_i}{2T}v^2\sin\theta=\mathrm{St}(h). \tag{6}$$

Transforming to the ballooning representation, and using (2), (3), and (5), we then find

$$\sigma=t^2\frac{\omega^2}{\omega_A^2}\Lambda\xi, \tag{7}$$

where (cf. Ref. 2)

$$\Lambda=-\frac{1}{2\omega^2R^2q^2n_0}\left(\int v^2Fh\,d\mathbf{v}\sin\theta\right)^{(\theta)}. \tag{8}$$

We see that the effects of the collisional and resonant interaction of ions with drift-ballooning modes lead to a renormalization of the inertial term in Eq. (1) (cf. Refs. 2 and 5). Making use of this circumstance, and also using the procedure of joining the asymptotic solutions of Eq. (1) with those of Eq. (1) in the "inertialess" region,[4] we find the dispersion relation [cf. Eq. (26) of Ref. 6]

$$\omega^2(1-\omega_{pi}^*/\omega+\Lambda)=-\gamma_0^2, \tag{9}$$

where γ_0 is the growth rate of the drift-ballooning modes, given explicitly in Ref. 4.

The problem has now been reduced to one of calculating the quantity λ in the various regimes from (6) and (8) and solving the dispersion relation (9).

3. SOLUTION OF THE DISPERSION

3.1. Resonant interaction. In accordance with the Introduction, we consider two cases of the resonant interaction of wave with untrapped ions. The first case deals with the region of perturbation frequencies[2] $\varepsilon^{1/2}v_{Ti}/qR<\omega<v_{Ti}/qR$; the second deals with perturbations with frequencies[3] $\omega<\varepsilon^{1/2}v_{Ti}/qR$.

3.1.1. The case of frequencies $\varepsilon^{1/2}v_{Ti}/qR<\omega<v_{Ti}/qR$. We solve kinetic Equation (6) as in Ref. 2. Incorporating drift effects, we find an e
for Λ:

$$\Lambda=i\frac{\sqrt{\pi}}{2}\frac{qv_{Ti}}{R\omega}\left[1-\frac{\omega_{ni}^*}{\omega}\left(1+\frac{3}{2}\eta\right)\right].$$

Let us consider the very simple case γ_0
This condition means that the MHD insta
pressed by the effects of the finite ion
us. From (9) and (10) we then find

$$\omega=\omega_{ni}^*\left(1+\frac{3}{2}\eta\right)+i\sqrt{\pi}\frac{\omega_{ni}^{*2}R}{qv_{Ti}}\left(1+\frac{3}{2}\eta\right)\eta.$$

An instability occurs if

$$\eta>0 \text{ and } \eta<-{}^2/_3.$$

The ratio of the imaginary and real part frequency is $\sim\omega_{ni}^*R/qv_{Ti}$.

3.1.2. The case of frequencies $\omega<$
In this case, making use of results from
find the following expression for Λ:

$$\Lambda=\frac{3q^2}{8\sqrt{2\varepsilon}}\lambda,$$

where

$$\lambda=1-\frac{\omega_{pi}^*}{\omega}+i\frac{\sqrt{\pi}}{18\sqrt{2}}\left(\frac{\omega}{\omega_b}\right)^5\left[1-\frac{\omega_{ni}^*}{\omega}\left(1-\frac{3}{2}\eta\right)\right.$$

By analogy with Ref. 3, we find two wa
in the case $\gamma_0\ll\omega_{pi}^*$:

$$\omega_1=\omega_{pi}^*-i\frac{\eta}{1+\eta}\frac{5\sqrt{\pi}}{36\sqrt{2}(1+8\sqrt{2\varepsilon}/3q^2)}\left(\frac{\omega_{pi}^*}{\omega_b}\right)^5\omega_p$$

$$\omega_2=\omega_2^*+i\frac{3\eta-2}{1+\eta}\frac{\sqrt{\pi}}{36\sqrt{2}(1+8\sqrt{2\varepsilon}/3q^2)}\left(\frac{\omega_2^*}{\omega_b}\right)^5\omega$$

where

$$\omega_2^*=\frac{\gamma_0^2}{\omega_{pi}^*(1+3q^2/8\sqrt{2\varepsilon})}.$$

The first branch is unstable if

$$-1<\eta<0,$$

and the second if

$$\eta>{}^2/_3 \text{ and } \eta<-1.$$

The ratio of the growth rate to the real the frequency is $\sim(\omega_{pi}^*/\omega_b)^5$.

Expression (15) for the frequency to that given in Ref. 3 but refers to no Mercier perturbations but also ballooning turbations. In the case in which γ_0 is rate of Mercier perturbations, expressio duces to that derived in Ref. 3. The b mode growth rate, can also be substitut as γ_0.

3.2. Collisional interaction. Agai regime we will discuss two cases: that o collisions and that of frequent collisions.

Ref. 2, the transition between these two cases is $\omega \approx \nu_i/\varepsilon$. The methods for solving the kinetic uation and for calculating Λ are described in tail in Ref. 2 (see also Ref. 7). We will reproduce ly the results here.

The quantity Λ is given by

$$\Lambda=-i\frac{\sqrt{2}\,q^2}{\varepsilon^2 n_0\omega}\frac{M_i}{T}\int_0^\infty FE^{3/2}\left(1-\frac{\hat{\omega}^*}{\omega}\right)dE\int\frac{d\lambda}{\lambda}\int d\theta\{[(1-\lambda B)^{1/2}+Y] -i\omega[(1-\lambda B)^{1/2}+Y]\}. \qquad (19)$$

ere $\lambda = \mu/E$, $\mu = v_\perp^2/2B$, $v_\perp$ is the modulus of e transverse velocity of the particles, and B is e equilibrium magnetic field. The integration over is from 0 to $1/B$; the region $0 < \lambda < 1/B_{max}$ corresponds to untrapped particles, and the region $1/B_{max} < \lambda < 1/B_{min}$ to trapped particles. The integration over θ is from 0 to 2π in the case of untrapped particles and between the turning points in e case of the trapped particles. The function $Y = Y(\lambda, E)$ is nonzero only for the untrapped particles. satisfies the relation

$$\int_0^{2\pi}\frac{d\theta}{(1-\lambda B)^{1/2}}\{\mathrm{St}[(1-\lambda B)^{1/2}+Y]+i\omega[(1-\lambda B)^{1/2}+Y]\}=0. \qquad (20)$$

he collision operator is[8]

$$\mathrm{St}(f)=2\bar{\nu}_i\left(\frac{T}{M_iE}\right)^{3/2}\frac{(1-\lambda B)^{1/2}}{B}\frac{\partial}{\partial\lambda}\left[\lambda(1-\lambda B)^{1/2}\frac{\partial f}{\partial\lambda}\right], \qquad (21)$$

here

$$\bar{\nu}_i=3\sqrt{\pi}\nu_i H(z)/2^{3/2},\quad z=(M_iE/T)^{1/2},$$

$$H(z)=\frac{1}{\sqrt{\pi}\,z}\exp(-z^2)+\left(1-\frac{1}{2z^2}\right)\frac{2}{\sqrt{\pi}}\int_0^z\exp(-t^2)\,dt. \qquad (22)$$

3.2.1. The case of infrequent collisions, $\omega > \nu_i/\varepsilon$. Ignoring the collision term in Eq. (20), we find the following expression for the function Y:

$$Y=Y_0=-\frac{\pi}{\sqrt{2}}\frac{\varkappa\varepsilon^{1/2}}{K(1/\varkappa)}, \qquad (23)$$

where $\varkappa=(2\varepsilon)^{-1/2}(1/\lambda B-1-\varepsilon)^{1/2}$. In this case Λ becomes

$$\Lambda=\tau(1-\omega_{pi}^*/\omega), \qquad (24)$$

where

$$\tau=\frac{2^{7/2}q^2}{3\pi\varepsilon^{1/2}}\left\{1+\frac{9}{2}\int_1^\infty\varkappa^2\,d\varkappa\left[E\left(\frac{1}{\varkappa}\right)-\frac{\pi^2}{4K(1/\varkappa)}\right]\right\}. \qquad (25)$$

Here $K(1/\kappa)$ and $E(1/\kappa)$ are complete elliptic integrals of the first and second kinds, respectively. Now assuming that ν_i is small but nonzero, we can incorporate collisions in Eq. (20). For this purpose we use the method of Ref. 8. We write the function Y in the form $Y = Y_0(1 + y)$; from Eq. (22) we then find

$$\frac{d^2y}{dx^2}+i\frac{\varepsilon\omega_0}{2\bar{\nu}_i}\left(\frac{EM_i}{T}\right)^{3/2}\ln\left(\frac{16}{|x|}\right)y=0. \qquad (26)$$

Here ω_0 is the real part of the frequency, and the parameter x is related to λ by $\lambda = [1 + \varepsilon(2x - 1)]/$ B. We seek the function y by making use of boundary conditions $y = -1$ at $x = 0$ and $y \to$ $x \to -\infty$. From Eq. (26) we then find, in th quasiclassical approximation,

$$y=-\exp\left[\int_0^x\sigma(x')\,dx'\right],$$

where

$$\sigma(x)=\frac{1-i}{\sqrt{2}}\left[\frac{\varepsilon\omega_0}{2\bar{\nu}_i}\left(\frac{M_iE}{T}\right)^{3/2}\ln\left(\frac{16}{|x|}\right)\right]^{1/2}.$$

Using expressions (19), (23), (27) and (28), find

$$\Lambda=\Lambda_0+\Lambda_1,$$

where

$$\Lambda_1=-(1+i)\mu\left(\frac{\nu_i}{\varepsilon\omega_0}\right)^{1/2}\left[1-\frac{\omega_{pi}^*}{\omega}+\eta\frac{\omega_{ni}^*}{\omega}\left(\frac{5}{2}-\frac{I_2}{I_1}\right)\right].$$

Here

$$\mu=\frac{\sqrt{3}\,\pi^{3/2}q^2I_1}{2^{7/4}\varepsilon^{1/2}\ln^{1/2}(128\varepsilon\omega_0/\nu_i)},$$

and the numbers I_1, and I_2 are defined by

$$I_1=\int_0^\infty H^{1/2}(z)z^{5/2}\exp(-z^2)\,dz^2,\quad I_2=\int_0^\infty H^{1/2}(z)z^{7/2}\exp(-z$$

An evaluation of these integrals yields $I_1 = 1$ and $I_2 = 0.74$.

Assuming $\gamma_0 \ll \omega_{pi}^*$, we find from (9), (30), and (31)

$$\omega=\omega_{pi}^*+1{,}97i\left(\frac{\nu_i}{\varepsilon\omega_{pi}^*}\right)^{1/2}\frac{\mu\eta}{1+\tau}\omega_{ni}^*.$$

We see that an instability occurs in the case The ratio of the growth rate of the frequency $\sim(\nu_i/\varepsilon\omega_{pi}^*)^{1/2}$.

Along with (33), the dispersion relation solution with $\omega < \omega_{pi}^*$. In this case we have

$$\omega=\omega_0+i(\nu_i\omega_0/\varepsilon)^{1/2}\frac{\mu}{1+\tau}\frac{1-0{,}97\eta}{1+\eta},$$

where

$$\omega_0=\gamma_0^2/\omega_{pi}^*(1+\tau).$$

An instability occurs if $-1 < \eta < 1.03$. The r of the growth rate to the frequency is $\sim(\nu_i/\varepsilon$

3.2.2. The case of frequent collisions,

Ignoring the term with ω in Eq. (20), we fin

$$\frac{1}{B}\frac{\partial Y}{\partial\lambda}=\frac{\pi}{2^{7/2}}\frac{1}{\varkappa E(1/\varkappa)\varepsilon^{1/2}}.$$

From (19) and (36) we then find

$$\Lambda=i\frac{q^2\pi}{4\varepsilon^{3/2}}\frac{3}{2}\frac{\nu_i}{\omega}\left\{[\sqrt{2}-\ln(1+\sqrt{2})]-\frac{\omega_{ni}^*}{\omega}\left[\left(1-\frac{3}{2}\eta\right)\right.\right.$$

$$-\ln(1+\sqrt{2})]+\frac{1}{\sqrt{2}}\eta\Big]\Big\}\Big\{1-2\int_1^\infty d\varkappa\Big[\frac{1}{E(1/\varkappa)}-\frac{4}{\pi^2}K\Big(\frac{1}{\varkappa}\Big)\Big]\Big\}. \quad (37)$$

From (9) and (37) we find the wave branch

$$\omega=\omega_{*i}^{*}[1+\eta(1-k)]-i2\frac{\varepsilon^{3/2}\omega_{*i}^{*2}}{q^2\nu_i}k\eta[1+\eta(1-k)]. \quad (38)$$

Here $k \simeq 1.03$. We see that this branch is unstable if $\eta < 0$.

4. DISCUSSION OF RESULTS

This analysis indicates the possibility of nonhydromagnetic instabilities of drift-ballooning modes as a result of resonant and collisional interactions of waves with ions. Correspondingly, these instabilities could be called "resonant" and "collisional." The resonant instabilities are characterized by (11), (15), and (16); the collisional instabilities by (33), (34), and (38). Both depend strongly on the parameter $\eta = \partial \ln T/\partial \ln n_0$ [see, e.g., conditions (12), (17) and (18)].

These instabilities may be of interest for the theory of plasma turbulence in tokamaks, in particular, because they may — by virtue of the nonlinearity of the plasma equations — influence the longer-wavelength (hydromagnetically stable) perturbations. Furthermore, hydromagnetic stability can be achieved not only by virtue of the finite-Larmor-radius effects but also by imposing an external agent on the plasma, e.g., by in
particles into the tokamak.[9] Under thes
the instabilities which we have been dis
may be the most important instabilities.
these arguments will of course require a
tailed theory, but that would go beyond
of the present paper.

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